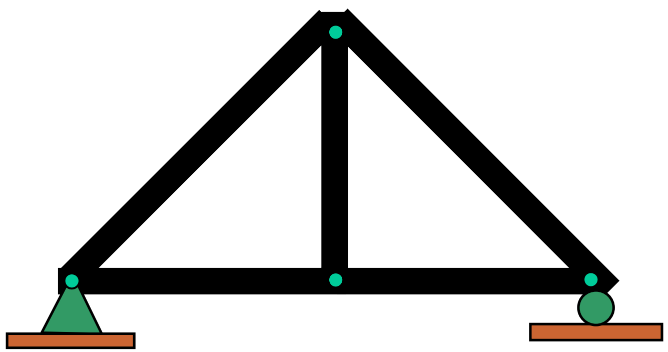
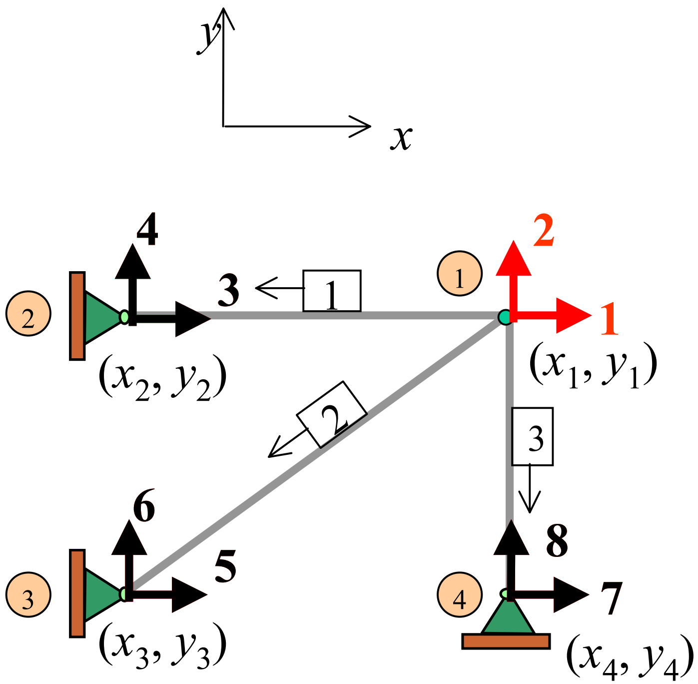
TRUSSES ANALYSIS

- Fundamentals of the Stiffness Method
- Member Local Stiffness Matrix
- Displacement and Force Transformation Matrices
- Member Global Stiffness Matrix
- Application of the Stiffness Method for Truss Analysis
- Trusses Having Inclined Supports, Thermal Changes and Fabrication Errors
- Space-Truss Analysis

2-Dimension Trusses



Fundamentals of the Stiffness Method



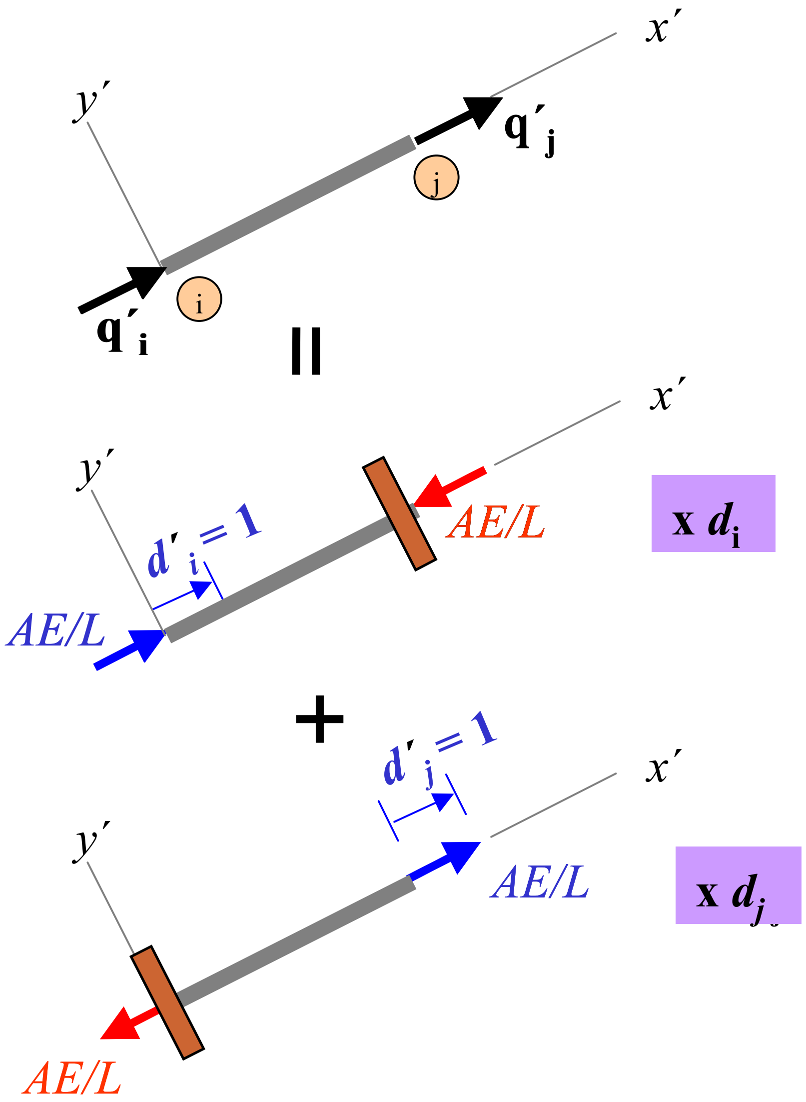
- **Node and Member Identification**

- **Global and Member Coordinates**

- **Degrees of Freedom**

  - **Known degrees of freedom D_3, D_4, D_5, D_6, D_7 and D_8**
  - **Unknown degrees of freedom D_1 and D_2**

Member Local Stiffness Matrix



$$q'_i = \frac{AE}{L}d'_i - \frac{AE}{L}d'_j$$

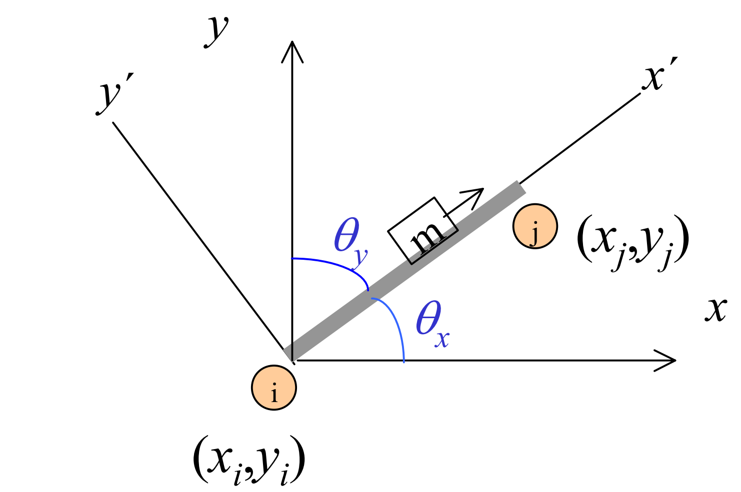
$$q'_j = -\frac{AE}{L}d'_i + \frac{AE}{L}d'_j$$

$$\begin{bmatrix} q'_i \\ q'_j \end{bmatrix} = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} d'_i \\ d'_j \end{bmatrix}$$

$$\boxed{[q'] = [k'][d']} \text{ ----------(1)}$$

$$[k'] = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

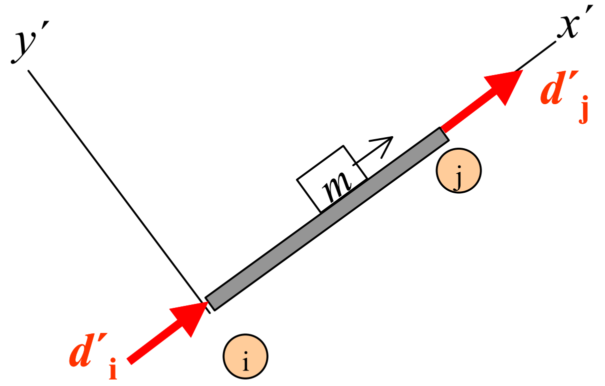
Displacement and Force Transformation Matrices



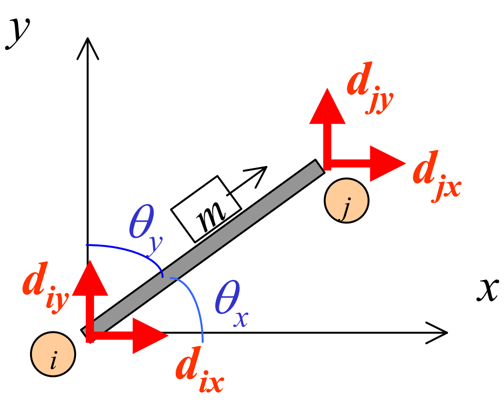
$$\lambda_x = \cos\theta_x = \frac{x_j - x_i}{L} = \frac{x_j - x_i}{\sqrt{(x_j - x_i)^2 + (y_j - y_i)^2}}$$

$$\lambda_y = \cos\theta_y = \frac{y_j - y_i}{L} = \frac{y_j - y_i}{\sqrt{(x_j - x_i)^2 + (y_j - y_i)^2}}$$

- **Displacement Transformation Matrices**



Local



Global

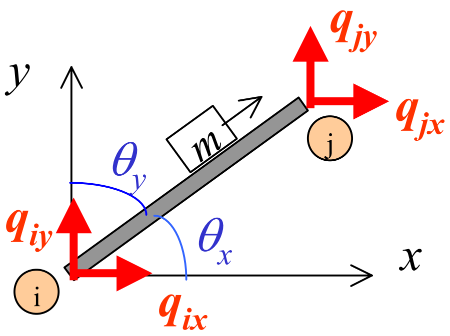
$$d'_i = d_{ix}\overset{\lambda_x}{\cos\theta_x} + d_{iy}\overset{\lambda_y}{\cos\theta_y}$$

$$d'_j = d_{jx}\cos\theta_x + d_{jy}\cos\theta_y$$

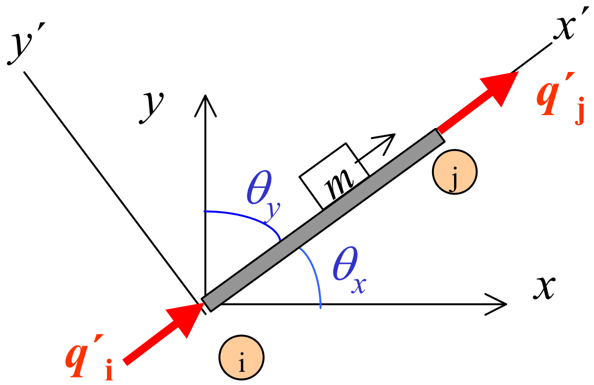
$$\begin{bmatrix} d'_i \\ d'_j \end{bmatrix} = \begin{bmatrix} \lambda_x & \lambda_y & 0 & 0 \\ 0 & 0 & \lambda_x & \lambda_y \end{bmatrix} \begin{bmatrix} d_{ix} \\ d_{iy} \\ d_{jx} \\ d_{jy} \end{bmatrix} \qquad [T] = \begin{bmatrix} \lambda_x & \lambda_y & 0 & 0 \\ 0 & 0 & \lambda_x & \lambda_y \end{bmatrix}$$

$$\boxed{[\boldsymbol{d'}] = [\boldsymbol{T}][\boldsymbol{d}]} \qquad \text{----------}(2)$$

• **Force Transformation Matrices**



Global



Local

$$q_{ix} = q'_i \overbrace{\cos\theta_x}^{\lambda_x}$$

$$q_{iy} = q'_i \overbrace{\cos\theta_y}^{\lambda_y}$$

$$q_{jx} = q'_j \cos\theta_x$$

$$q_{jy} = q'_j \cos\theta_y$$

$$\begin{bmatrix} q_{ix} \\ q_{iy} \\ q_{jx} \\ q_{jy} \end{bmatrix} = \begin{bmatrix} \lambda_x & 0 \\ \lambda_y & 0 \\ 0 & \lambda_x \\ 0 & \lambda_y \end{bmatrix} \begin{bmatrix} q'_i \\ q'_j \end{bmatrix}$$

where

$$[T]^T = \begin{bmatrix} \lambda_x & 0 \\ \lambda_y & 0 \\ 0 & \lambda_x \\ 0 & \lambda_y \end{bmatrix}$$

$$\boxed{[q] = [T]^{T}[q']} \quad \text{----------(3)}$$

Member Global Stiffness Matrix

$$\boxed{[q] = [T]^T[q']}$$ ----------(3)

Substitute ($[q'] = [k'][\mathbf{d}'] + [q'^F]$) into Eq. 3, yields the result,

$$[q] = \underbrace{[T]^T([k'][d']}_{[k]} + [q'^F]) = [T]^T[k'][T][d] + [T]^T[q'^F] = [k][d] + [q^F]$$

$$[k] = [T]^T[k'][T]$$

$$[q^F] = [T]^T[q'^F]$$

$$[k] = \begin{pmatrix} \lambda_x & 0 \\ \lambda_y & 0 \\ 0 & \lambda_x \\ 0 & \lambda_y \end{pmatrix} \frac{AE}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \lambda_x & \lambda_y & 0 & 0 \\ 0 & 0 & \lambda_x & \lambda_y \end{bmatrix}$$

$$[k] = \frac{AE}{L} \begin{array}{c} \\ U \\ V \\ U \\ V \end{array} \begin{array}{c} \begin{array}{cccc} U & V & U & V \end{array} \\ \begin{pmatrix} \lambda_x\lambda_x & \lambda_x\lambda_y & -\lambda_x\lambda_x & -\lambda_x\lambda_y \\ \lambda_y\lambda_x & \lambda_y\lambda_y & -\lambda_y\lambda_x & -\lambda_y\lambda_y \\ -\lambda_x\lambda_x & -\lambda_x\lambda_y & \lambda_x\lambda_x & \lambda_x\lambda_y \\ -\lambda_y\lambda_x & -\lambda_y\lambda_y & \lambda_y\lambda_x & \lambda_y\lambda_y \end{pmatrix} \end{array}$$

Application of the Stiffness Method for Truss Analysis

Equilibrium Equation:

$$\boxed{[Q^a] = [K][D] + [Q^F]}$$

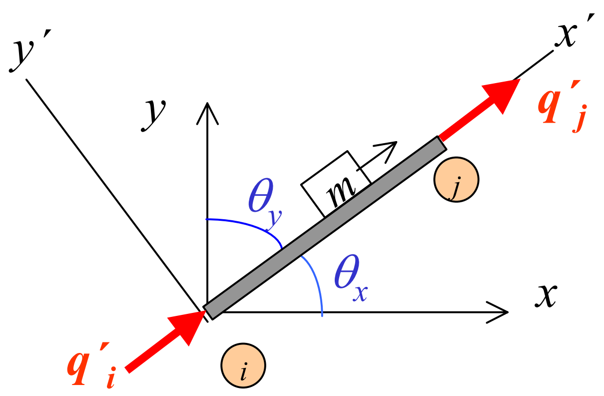
Partitioned Form:

$$\begin{array}{c} \text{Joint Load} \\ \begin{bmatrix} \mathbf{Q_k} \\ \mathbf{Q_u} \end{bmatrix} \\ \text{Reaction} \end{array} = \begin{bmatrix} \mathbf{K_{11}} & \mathbf{K_{12}} \\ \mathbf{K_{21}} & \mathbf{K_{22}} \end{bmatrix} \begin{array}{c} \text{Unknown Displacement} \\ \begin{bmatrix} \mathbf{D_u} \\ \mathbf{D_k} \end{bmatrix} \\ \text{Boundary Condition} \end{array} + \begin{bmatrix} \mathbf{Q^F{}_k} \\ \mathbf{Q^F{}_u} \end{bmatrix}$$

$$\mathbf{[Q_k] = [K_{11}][D_u] + [K_{12}][D_k] + [Q^F]}$$

$$\mathbf{[D_u] = [K_u]^{-1}\left(([Q_k] - [Q^F]) - [K_{12}][D_k]\right)}$$

▶ **Member Forces**



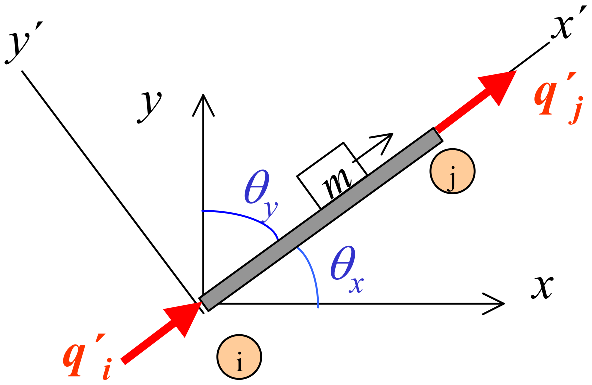
$$\begin{bmatrix} q'_i \\ q'_j \end{bmatrix} = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} d'_i \\ d'_j \end{bmatrix} + \begin{bmatrix} q'^F_i \\ q'^F_j \end{bmatrix}$$

$$\begin{bmatrix} \lambda_x & \lambda_y & 0 & 0 \\ 0 & 0 & \lambda_x & \lambda_y \end{bmatrix}\begin{bmatrix} d_{ix} \\ d_{iy} \\ d_{jx} \\ d_{jy} \end{bmatrix}$$

$$\begin{bmatrix} q'_i \\ q'_j \end{bmatrix} = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} \lambda_x & \lambda_y & 0 & 0 \\ 0 & 0 & \lambda_x & \lambda_y \end{bmatrix}\begin{bmatrix} D_{ix} \\ D_{iy} \\ D_{jx} \\ D_{jy} \end{bmatrix} + \begin{bmatrix} q'^F_i \\ q'^F_j \end{bmatrix}$$

$$\begin{bmatrix} q'_i \\ q'_j \end{bmatrix} = \frac{AE}{L}\begin{bmatrix} \lambda_x & \lambda_y & -\lambda_x & -\lambda_y \\ -\lambda_x & -\lambda_y & \lambda_x & \lambda_y \end{bmatrix}\begin{bmatrix} D_{ix} \\ D_{iy} \\ D_{jx} \\ D_{jy} \end{bmatrix} + \begin{bmatrix} q'^F{}_i \\ q'^F{}_j \end{bmatrix}$$

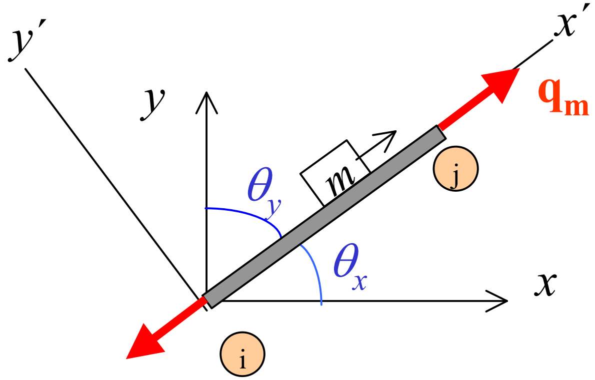
$$q'_j = \frac{AE}{L}\begin{bmatrix} -\lambda_x & -\lambda_y & \lambda_x & \lambda_y \end{bmatrix}\begin{pmatrix} D_{xi} \\ D_{yi} \\ D_{xj} \\ D_{yj} \end{pmatrix} + q_j'^F$$



Member Forces

▶ Member Forces

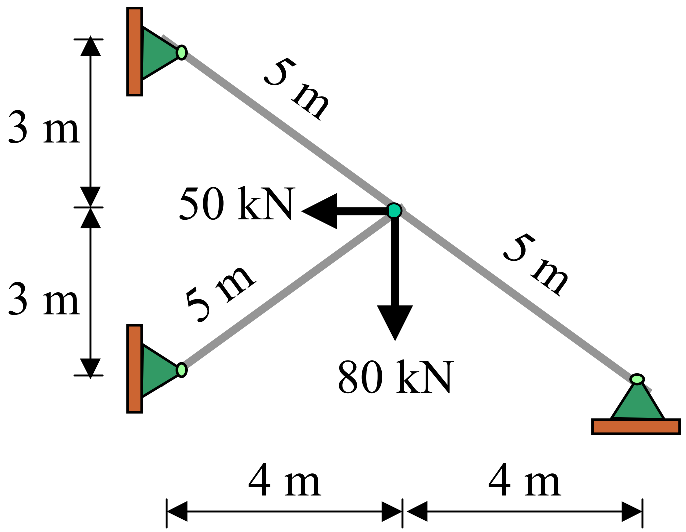
$$q_m = \frac{AE}{L}\begin{bmatrix} -\lambda_x & -\lambda_y & \lambda_x & \lambda_y \end{bmatrix}\begin{pmatrix} D_{xi} \\ D_{yi} \\ D_{xj} \\ D_{yj} \end{pmatrix} + q_j'^F$$

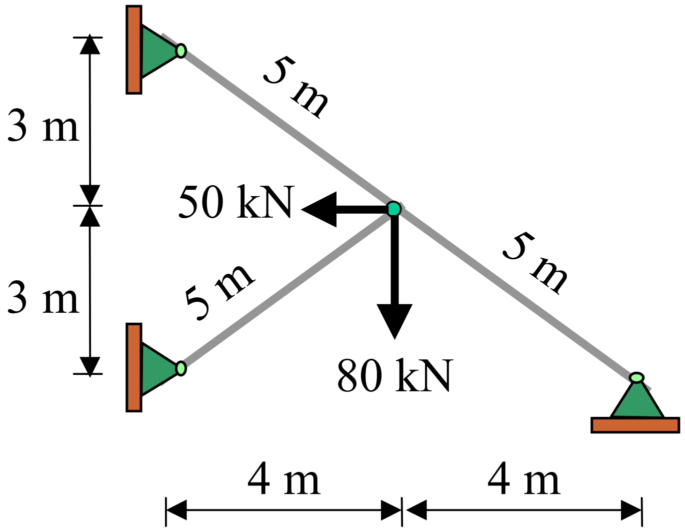


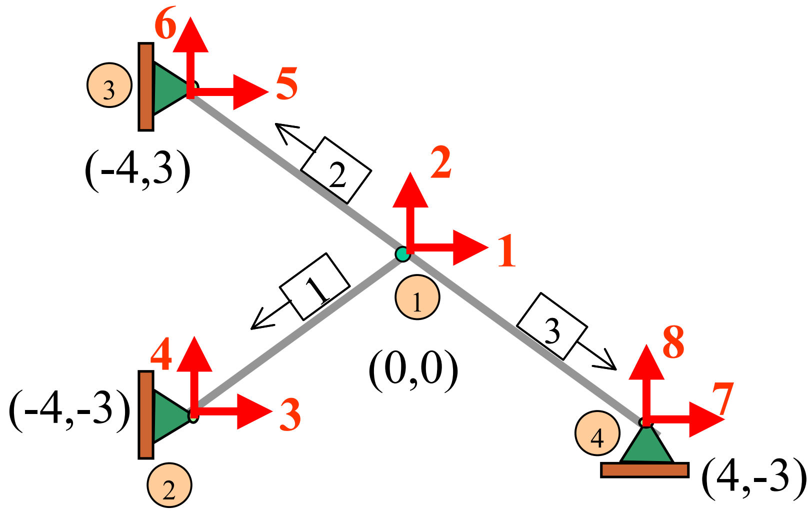
Member Forces

Example 1

For the truss shown, use the stiffness method to:
(a) Determine the **deflections** of the loaded joint.
(b) Determine the **end forces** of each member and **reactions** at supports.
Assume EA to be the same for each member.





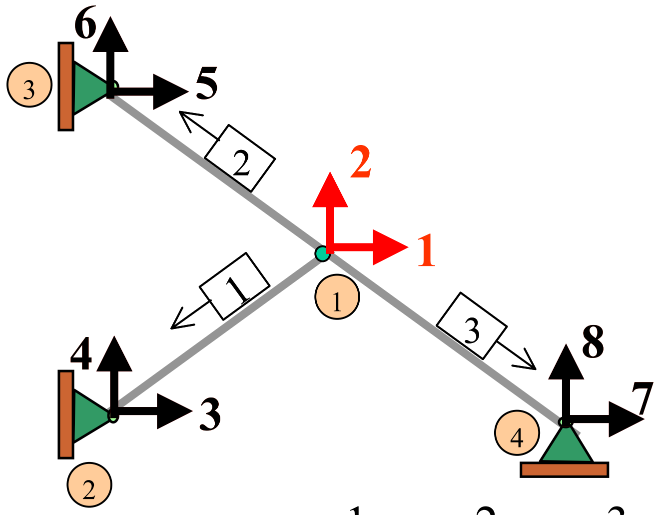


$$\hat{\lambda}_{ij} = \frac{(x_j - x_i)\hat{i}}{L} + \frac{(y_j - y_i)\hat{j}}{L}$$

$cos\,\theta_x = \lambda_x$ $\quad$ $cos\,\theta_y = \lambda_y$

$$[\,k\,]_m = \frac{AE}{L}\begin{array}{c|cccc} & U_i & V_i & U_j & V_j \\ \hline U_i & \lambda_x\lambda_x & \lambda_x\lambda_y & -\lambda_x\lambda_x & -\lambda_x\lambda_y \\ V_i & \lambda_y\lambda_x & \lambda_y\lambda_y & -\lambda_y\lambda_x & -\lambda_y\lambda_y \\ U_j & -\lambda_x\lambda_x & -\lambda_x\lambda_y & \lambda_x\lambda_x & \lambda_x\lambda_y \\ V_j & -\lambda_y\lambda_x & -\lambda_y\lambda_y & \lambda_y\lambda_x & \lambda_y\lambda_y \end{array}$$

Member	λ_x	λ_y
#1	-4/5 = -0.8	-3/5 = -0.6
#2	-4/5 = -0.8	3/5 = 0.6
#3	4/5 = 0.8	-3/5 = -0.6



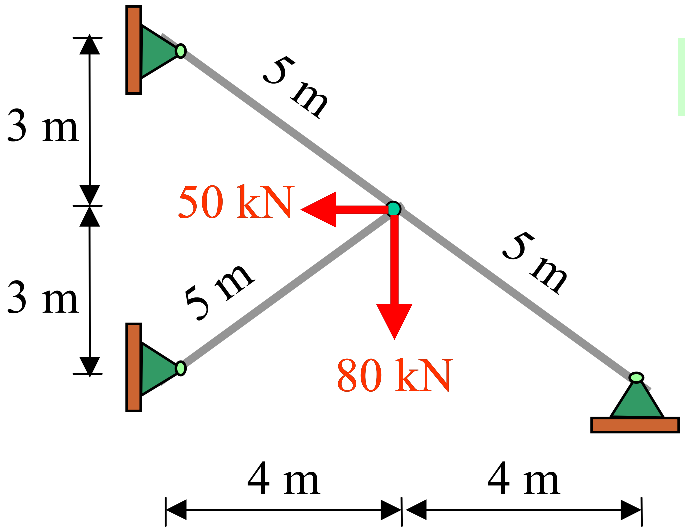
Member	λ_x	λ_y	λ_x^2	$\lambda_x\lambda_y$	λ_y^2
#1	-0.8	-0.6	0.64	0.48	0.36
#2	- 0.8	0.6	0.64	-0.48	0.36
#3	0.8	-0.6	0.64	-0.48	0.36

$$[k]_1 = \frac{AE}{5} \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} 0.64 & 0.48 & -0.64 & -0.48 \\ 0.48 & 0.36 & -0.48 & -0.36 \\ -0.64 & -0.48 & 0.64 & 0.48 \\ -0.48 & -0.36 & 0.48 & 0.36 \end{pmatrix} \end{array}$$

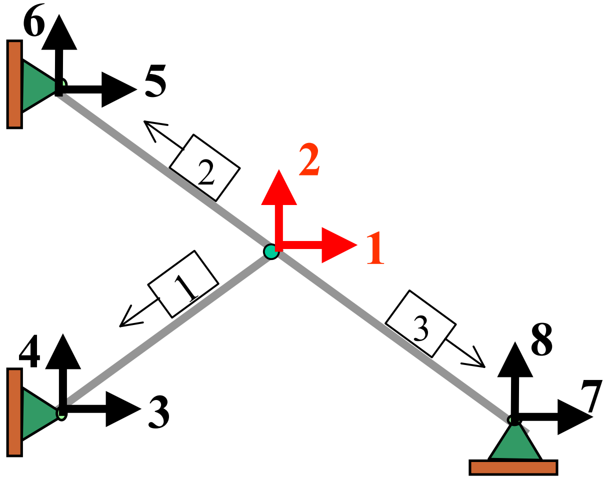
$$[k]_2 = \frac{AE}{5} \begin{array}{c} \\ 1 \\ 2 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 5 & 6 \end{array} \\ \begin{pmatrix} 0.64 & -0.48 & -0.64 & 0.48 \\ -0.48 & 0.36 & 0.48 & -0.36 \\ -0.64 & 0.48 & 0.64 & -0.48 \\ 0.48 & -0.36 & -0.48 & 0.36 \end{pmatrix} \end{array}$$

$$[k]_3 = \frac{AE}{5} \begin{array}{c} \\ 1 \\ 2 \\ 7 \\ 8 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 7 & 8 \end{array} \\ \begin{pmatrix} 0.64 & -0.48 & -0.64 & 0.48 \\ -0.48 & 0.36 & 0.48 & -0.36 \\ -0.64 & 0.48 & 0.64 & -0.48 \\ 0.48 & -0.36 & -0.48 & 0.36 \end{pmatrix} \end{array}$$

$$[K] = \frac{AE}{5} \begin{array}{c} \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{pmatrix} 1.92 & -0.48 \\ -0.48 & 1.08 \end{pmatrix} \end{array}$$

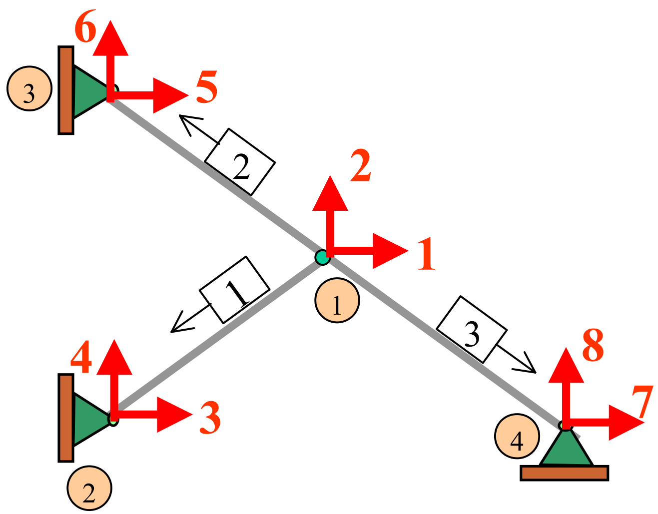


Global

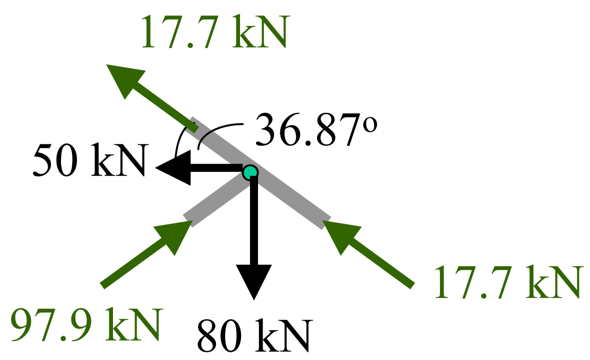


$$\begin{array}{c} \\ \left(\begin{array}{c} Q_1 = -50 \\ Q_2 = -80 \end{array}\right) \end{array} = \frac{AE}{5} \begin{array}{c} \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \left(\begin{array}{cc} 1.92 & -0.48 \\ -0.48 & 1.08 \end{array}\right) \end{array} \begin{array}{c} \\ \left(\begin{array}{c} D_1 \\ D_2 \end{array}\right) \end{array} + \begin{array}{c} \\ \left(\begin{array}{c} 0 \\ 0 \end{array}\right) \end{array} \begin{array}{c} \\ 1 \\ 2 \end{array}$$

$$\left(\begin{array}{c} D_1 \\ D_2 \end{array}\right) = \left(\begin{array}{c} -250.65/AE \\ -481.77/AE \end{array}\right)$$



Member	λ_x	λ_y
#1	-0.8	-0.6
#2	-0.8	0.6
#3	0.8	-0.6



Local

$$[q'_F]_m = \frac{AE}{L}\begin{bmatrix}-\lambda_x & -\lambda_y & \lambda_x & \lambda_x\end{bmatrix}\begin{bmatrix}D_{xi}\\ D_{yi}\\ D_{xj}\\ D_{yj}\end{bmatrix} + \left[q'^F\right]$$

$$[q'_F]_1 = \frac{AE}{5}\begin{bmatrix}0.8 & 0.6 & -0.8 & -0.6\end{bmatrix}\begin{pmatrix}D_1 = -250.65/AE\\ D_2 = -481.77/AE\\ D_3 = 0.0\\ D_4 = 0.0\end{pmatrix}$$

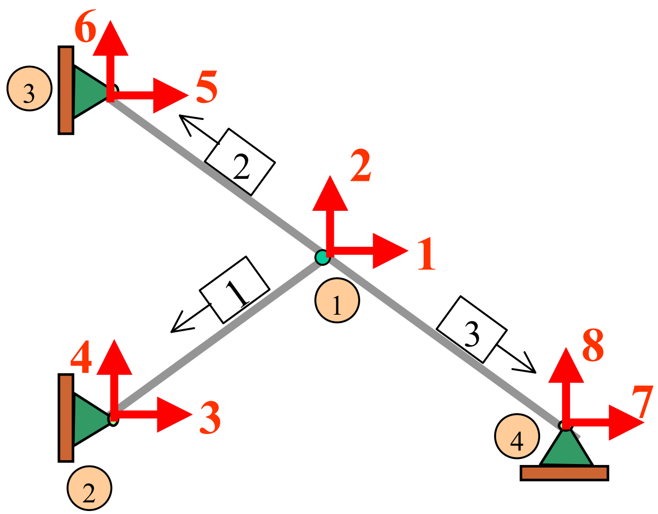
$$= -97.9 \text{ kN (C)}$$

$$[q'_F]_2 = \frac{AE}{5}\begin{bmatrix}0.8 & -0.6 & -0.8 & 0.6\end{bmatrix}\begin{pmatrix}D_1 = -250.65/AE\\ D_2 = -481.77/AE\\ D_5 = 0.0\\ D_6 = 0.0\end{pmatrix}$$

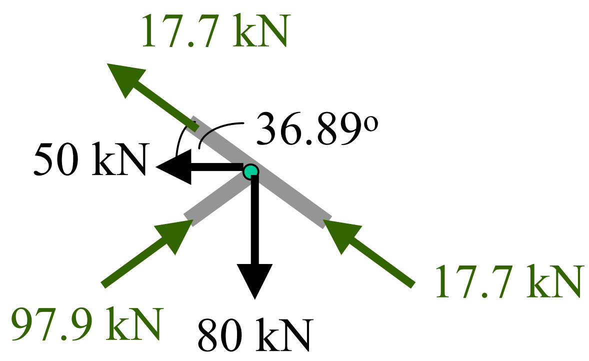
$$= +17.7 \text{ kN (T)}$$

$$[q'_F]_3 = \frac{AE}{5}\begin{bmatrix}-0.8 & +0.6 & +0.8 & -0.6\end{bmatrix}\begin{pmatrix}D_1 = -250.65/AE\\ D_2 = -481.77/AE\\ D_7 = 0.0\\ D_8 = 0.0\end{pmatrix}$$

$$= -17.7 \text{ kN (C)}$$

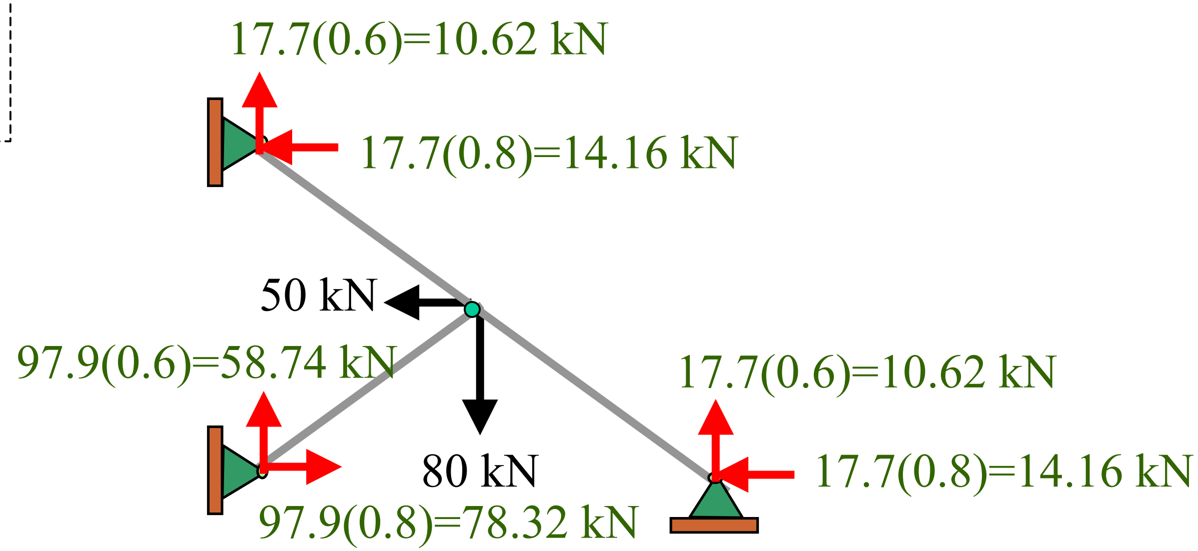


Member	λ_x	λ_y
#1	-0.8	-0.6
#2	-0.8	0.6
#3	0.8	-0.6



Check :

$$\nwarrow^{+} \quad \Sigma F_{x'} = 0: \quad 17.7 + 17.7 + 50\cos 36.89 - 97.9\cos 73.78 - 80\cos 53.11 = 0, \ O.K$$



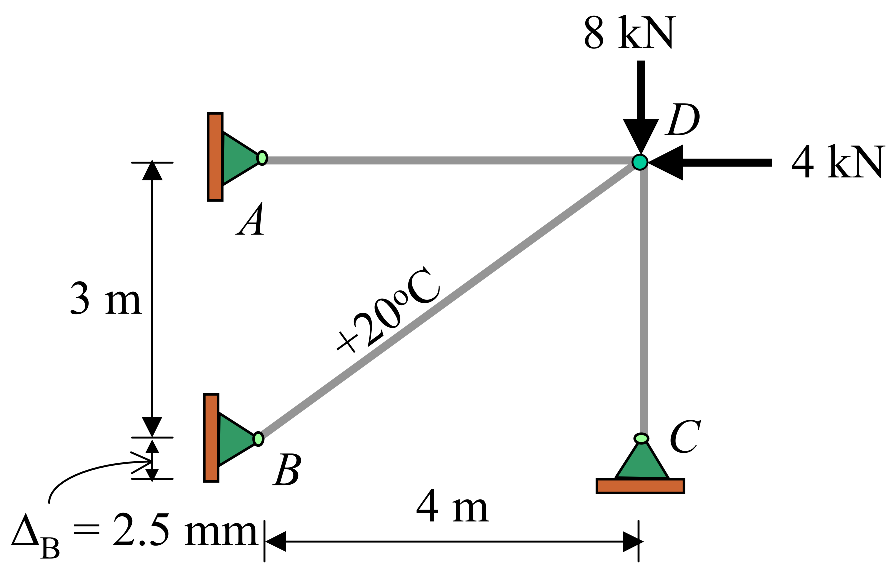
Example 2

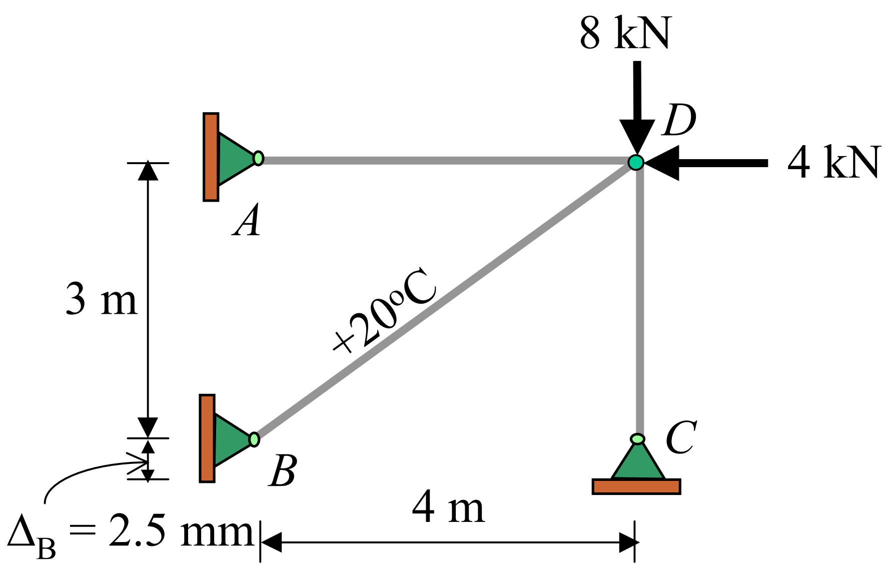
For the truss shown, use the stiffness method to:

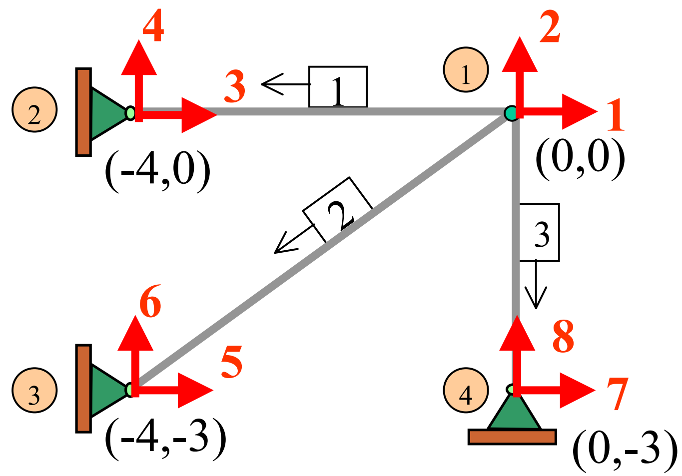
(b) Determine the **end forces** of each member and **reactions** at supports.

(a) Determine the **deflections** of the loaded joint.

The support *B* settles downward 2.5 mm. Temperature in member *BD* increase 20 °C. Take $\alpha = 12\text{x}10^{-6}$ /°C, $AE = 8(10^3)$ kN.





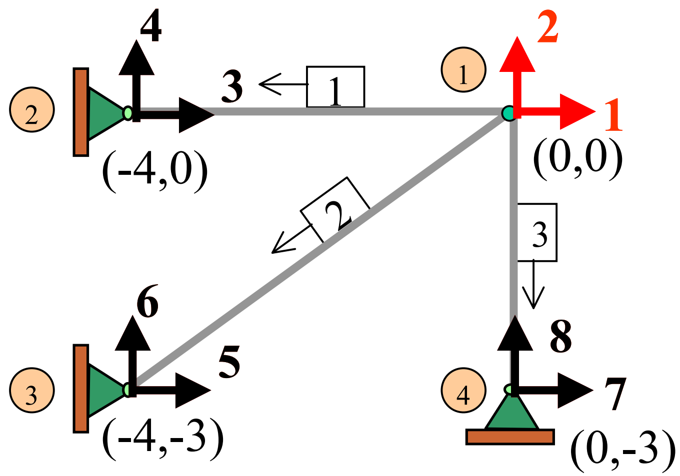


$$\hat{\lambda}_{ij} = \frac{(x_j - x_i)\hat{i}}{L} + \frac{(y_j - y_i)\hat{j}}{L}$$

$$cos\,\theta_x = \lambda_x \qquad cos\,\theta_y = \lambda_y$$

Member	λ_x	λ_y
#1	-4/4 = -1	0
#2	-4/5 = -0.8	-3/5 = -0.6
#3	0	-3/3 = -1

$$[\,k\,]_m = \frac{AE}{L} \begin{array}{c} \\ U_i \\ V_i \\ U_j \\ V_j \end{array} \begin{array}{c} \begin{array}{cccc} U_i & V_i & U_j & V_j \end{array} \\ \begin{pmatrix} \lambda_x\lambda_x & \lambda_x\lambda_y & -\lambda_x\lambda_x & -\lambda_x\lambda_y \\ \lambda_y\lambda_x & \lambda_y\lambda_y & -\lambda_y\lambda_x & -\lambda_y\lambda_y \\ -\lambda_x\lambda_x & -\lambda_x\lambda_y & \lambda_x\lambda_x & \lambda_x\lambda_y \\ -\lambda_y\lambda_x & -\lambda_y\lambda_y & \lambda_y\lambda_x & \lambda_y\lambda_y \end{pmatrix} \end{array}$$



Member	λ_x	λ_y	λ_x^2/L	$\lambda_x \lambda_y/L$	λ_y^2/L
#1	-1	0	0.25	0	0
#2	- 0.8	-0.6	0.128	0.096	0.072
#3	0	-1	0	0	0.333

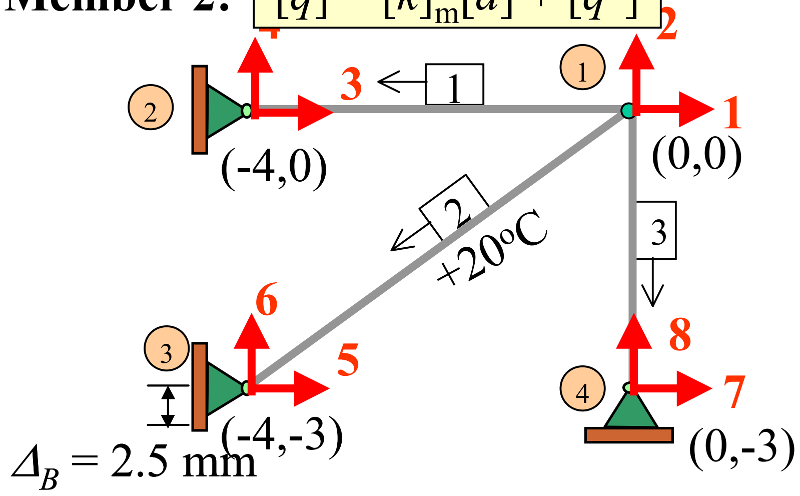
$$[k]_1 = 8\text{x}10^3 \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0.25 & 0 & -0.25 & 0 \\ 2 & 0 & 0 & 0 & 0 \\ 3 & -0.25 & 0 & 0.25 & 0 \\ 4 & 0 & 0 & 0 & 0 \end{array}$$

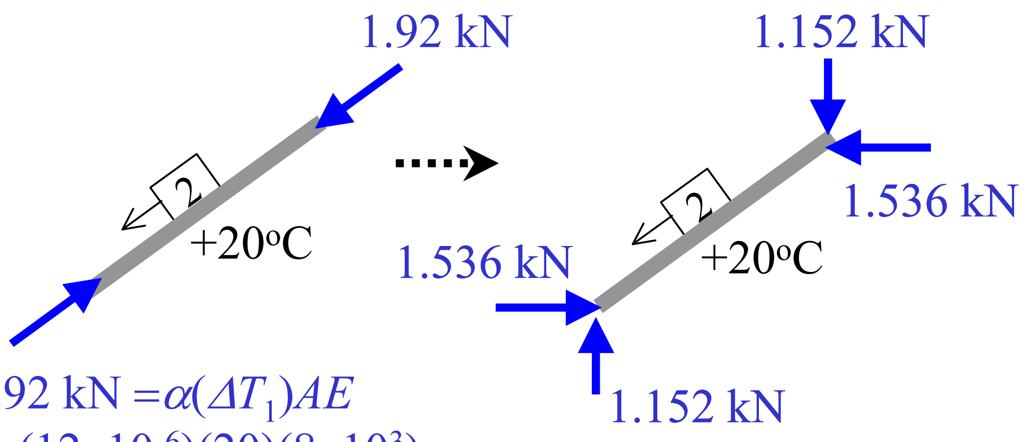
$$[k]_2 = 8\text{x}10^3 \begin{array}{c|cccc} & 1 & 2 & 5 & 6 \\ \hline 1 & 0.128 & 0.096 & -0.128 & -0.096 \\ 2 & 0.096 & 0.072 & -0.096 & -0.072 \\ 5 & -0.128 & -0.096 & 0.128 & 0.096 \\ 6 & -0.096 & -0.072 & 0.096 & 0.072 \end{array}$$

$$[k]_3 = 8\text{x}10^3 \begin{array}{c|cccc} & 1 & 2 & 7 & 8 \\ \hline 1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0.333 & 0 & -0.333 \\ 7 & 0 & 0 & 0 & 0 \\ 8 & 0 & -0.333 & 0 & 0.333 \end{array}$$

$$[K] = 8\text{x}10^3 \begin{array}{c|cc} & 1 & 2 \\ \hline 1 & 0.378 & 0.096 \\ 2 & 0.096 & 0.405 \end{array}$$

Member 2: $[q] = [k]_m[d] + [q^F]$



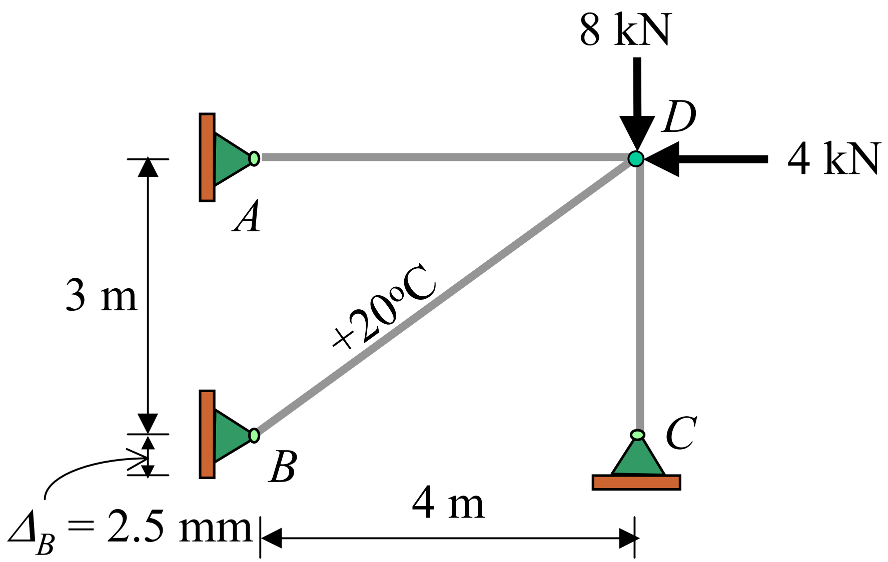


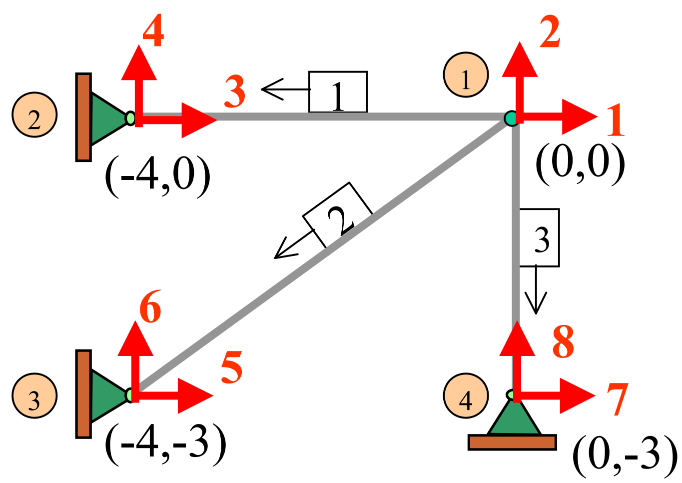
$1.92 \text{ kN} = \alpha(\Delta T_1)AE = (12\text{x}10^{-6})(20)(8\text{x}10^3)$

$$
\begin{pmatrix} q_1 \\ q_2 \\ q_5 \\ q_6 \end{pmatrix} = 8\text{x}10^3
\begin{array}{c} \\ 1 \\ 2 \\ 5 \\ 6 \end{array}
\begin{array}{c} \begin{array}{cccc} 1 & 2 & 5 & 6 \end{array} \\ \begin{pmatrix} 0.128 & 0.096 & -0.128 & -0.096 \\ 0.096 & 0.072 & -0.096 & -0.072 \\ -0.128 & -0.096 & 0.128 & 0.096 \\ -0.096 & -0.072 & 0.096 & 0.072 \end{pmatrix} \end{array}
\begin{pmatrix} d_1 \\ d_2 \\ d_5 = 0 \\ d_6 = -2.5\text{x}10^{-3} \end{pmatrix} +
\begin{pmatrix} -1.536 \\ -1.152 \\ 1.536 \\ 1.152 \end{pmatrix}
\begin{array}{c} 1 \\ 2 \\ 5 \\ 6 \end{array}
$$

$$
\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = 8\text{x}10^3
\begin{array}{c} \\ 1 \\ 2 \end{array}
\begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{pmatrix} 0.128 & 0.096 \\ 0.096 & 0.072 \end{pmatrix} \end{array}
\begin{pmatrix} d_1 \\ d_2 \end{pmatrix} + 8\text{x}10^3
\begin{array}{c} \begin{array}{cc} 5 & 6 \end{array} \\ \begin{pmatrix} -0.128 & -0.096 \\ -0.096 & -0.072 \end{pmatrix} \end{array}
\begin{pmatrix} 0 \\ -2.5\text{x}10^{-3} \end{pmatrix}
\begin{array}{c} 5 \\ 6 \end{array} +
\begin{pmatrix} -1.536 \\ -1.152 \end{pmatrix}
\begin{array}{c} 1 \\ 2 \end{array}
$$

$$
\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = 8\text{x}10^3
\begin{array}{c} 1 \\ 2 \end{array}
\begin{pmatrix} 0.128 & 0.096 \\ 0.096 & 0.072 \end{pmatrix}
\begin{pmatrix} d_1 \\ d_2 \end{pmatrix} +
\begin{pmatrix} 1.92 \\ 1.44 \end{pmatrix} +
\begin{pmatrix} -1.536 \\ -1.152 \end{pmatrix}
\begin{array}{c} 1 \\ 2 \end{array}
$$



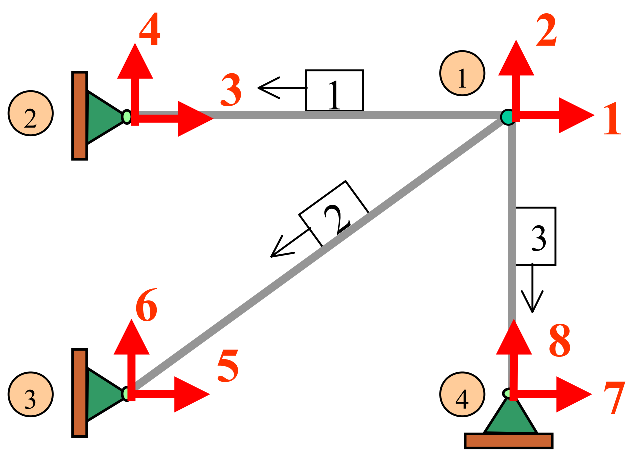


$$[Q] = [K][D] + [Q^F]$$

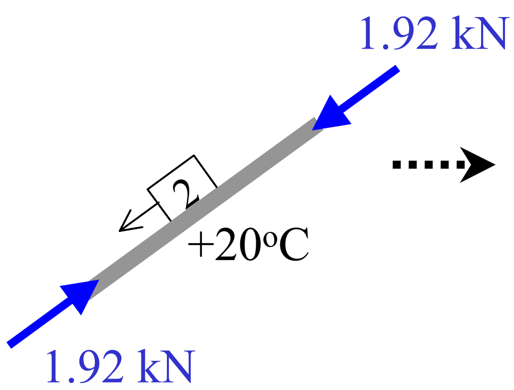
Global:

$$\begin{pmatrix} Q_1 = -4 \\ Q_2 = -8 \end{pmatrix} = 8\text{x}10^3 \begin{array}{c c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{pmatrix} 0.378 & 0.096 \\ 0.096 & 0.405 \end{pmatrix} \end{array} \begin{pmatrix} D_1 \\ D_2 \end{pmatrix} + \begin{pmatrix} 1.92 \\ 1.44 \end{pmatrix} + \begin{pmatrix} -1.536 \\ -1.152 \end{pmatrix}$$

$$\begin{pmatrix} D_1 \\ D_2 \end{pmatrix} = \begin{pmatrix} -0.8514\text{x}10^{-3} & \text{m} \\ -2.356\text{x}10^{-3} & \text{m} \end{pmatrix}$$



Member	λ_x	λ_y
#1	-1	0
#2	- 0.8	-0.6
#3	0	-1



Local

$$[q'_F]_m = \frac{AE}{L}\begin{bmatrix}-\lambda_x & -\lambda_y & \lambda_x & \lambda_x\end{bmatrix}\begin{bmatrix}D_{xi}\\ D_{yi}\\ D_{xj}\\ D_{yj}\end{bmatrix} + \begin{bmatrix}q'^F\end{bmatrix}$$

$$[q'_F]_1 = \frac{8\text{x}10^3}{4}\begin{bmatrix}1.0 & 0.0 & -1.0 & 0.0\end{bmatrix}\begin{pmatrix}D_1 = -0.8514\text{x}10^{-3}\\ D_2 = -2.356\text{x}10^{-3}\\ D_3 = 0.0\\ D_4 = 0.0\end{pmatrix}$$

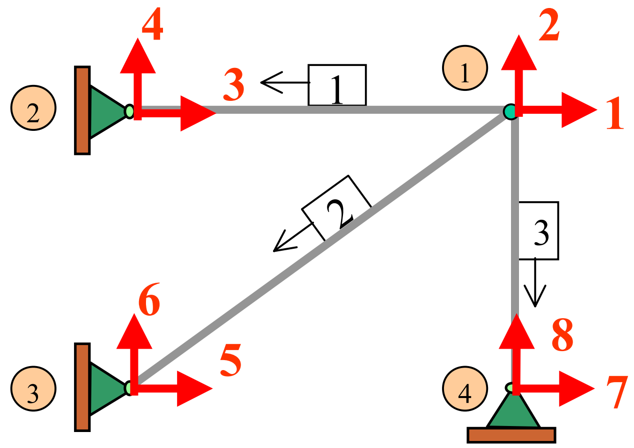
$= -1.70$ kN (C)

$$[q'_F]_2 = \frac{8\text{x}10^3}{5}\begin{bmatrix}0.8 & 0.6 & -0.8 & -0.6\end{bmatrix}\begin{pmatrix}D_1 = -0.8514\text{x}10^{-3}\\ D_2 = -2.356\text{x}10^{-3}\\ D_5 = 0.0\\ D_6 = -0.0025\end{pmatrix} + \begin{bmatrix}-1.92\end{bmatrix}$$

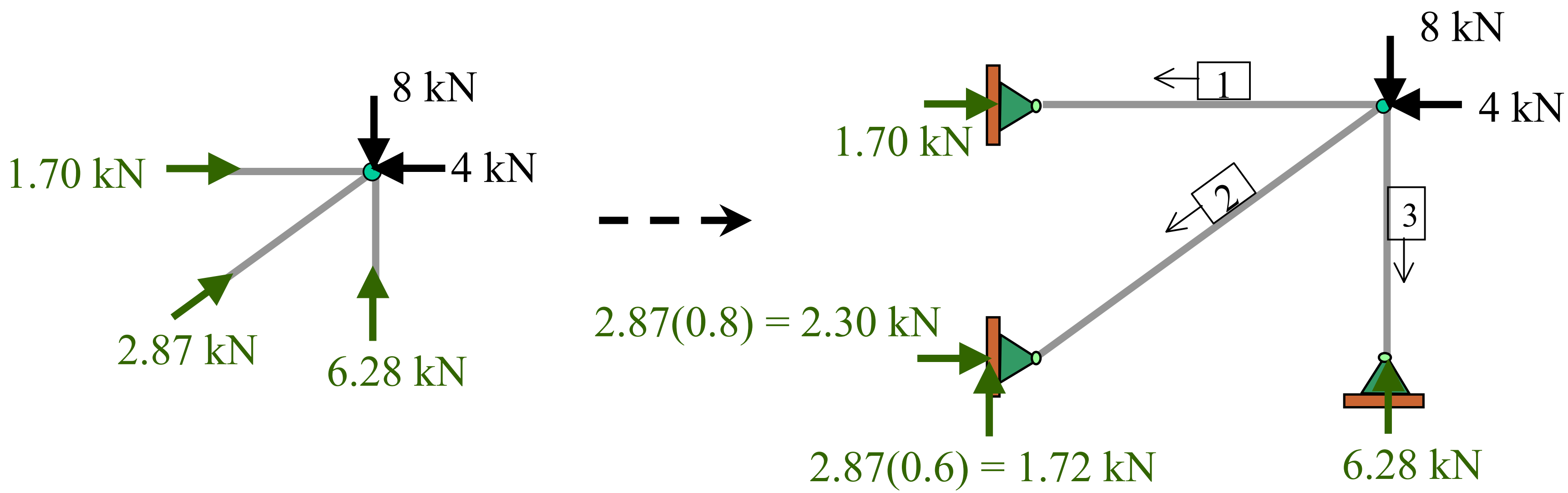
$= -2.87$ kN (C)

$$[q'_F]_3 = \frac{8\text{x}10^3}{3}\begin{bmatrix}0.0 & 1.0 & 0.0 & -1.0\end{bmatrix}\begin{pmatrix}D_1 = -0.8514\text{x}10^{-3}\\ D_2 = -2.356\text{x}10^{-3}\\ D_7 = 0.0\\ D_8 = 0.0\end{pmatrix}$$

$= -6.28$ kN (C)

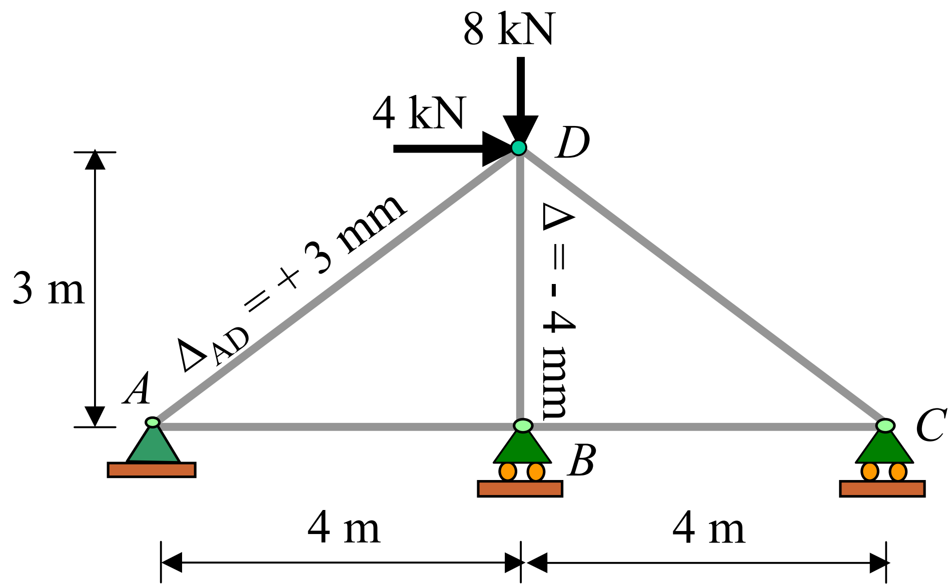


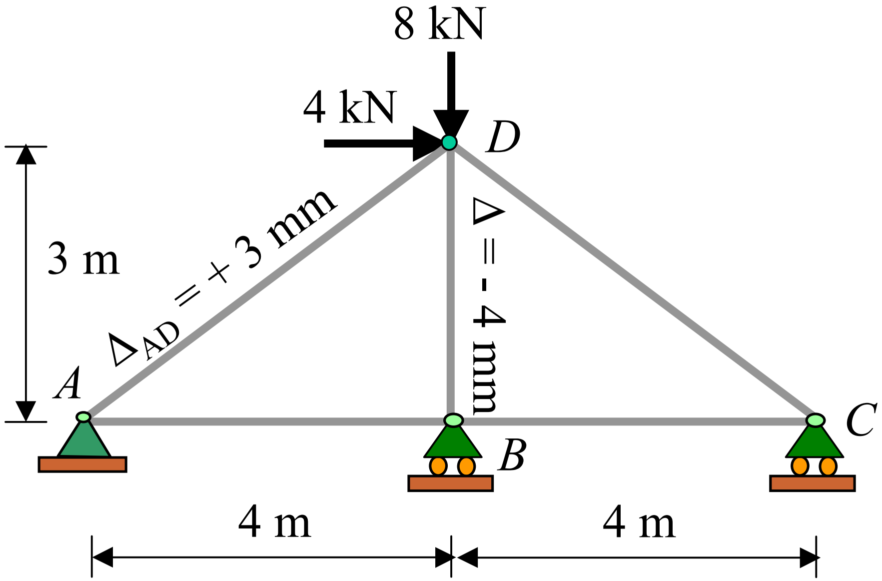
Member	$\cos\theta_x$	$\cos\theta_y$	$[q']_m$
#1	-1	0	-1.70
#2	- 0.8	-0.6	-2.87
#3	0	-1	-6.28

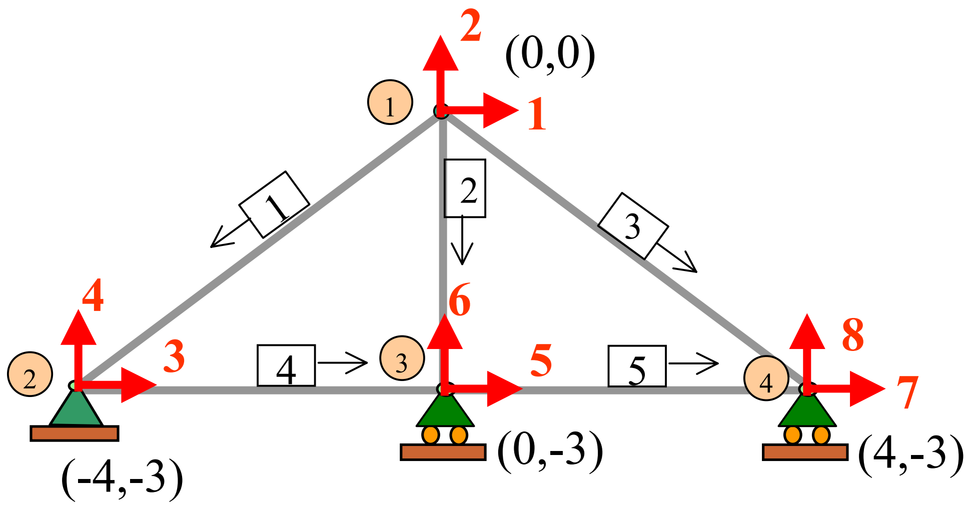


Example 3

For the truss shown, use the stiffness method to:
(a) Determine the **end forces** of each member and **reactions** at supports.
(b) Determine the **displacement** of the loaded joint.
Take $AE = 8(10^3)$ kN.





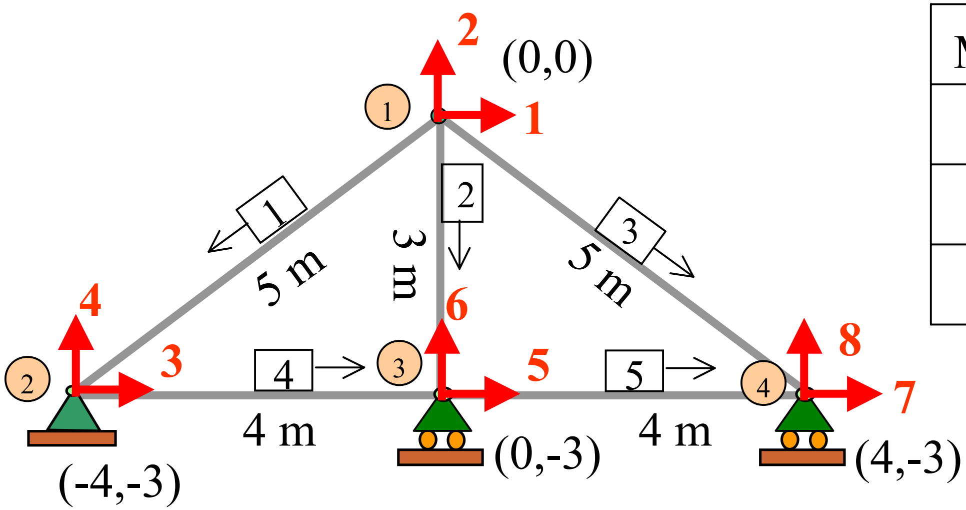


$$\hat{\lambda}_{ij} = \frac{(x_j - x_i)\hat{i}}{L} + \frac{(y_j - y_i)\hat{j}}{L}$$

$cos\theta_x = \lambda_x$ $\quad$ $cos\theta_y = \lambda_y$

$$[\ k\ ]_{\mathrm{m}} = \frac{AE}{L} \begin{array}{c|cccc} & U_i & V_i & U_j & V_j \\ \hline U_i & \lambda_x\lambda_x & \lambda_x\lambda_y & -\lambda_x\lambda_x & -\lambda_x\lambda_y \\ V_i & \lambda_y\lambda_x & \lambda_y\lambda_y & -\lambda_y\lambda_x & -\lambda_y\lambda_y \\ U_j & -\lambda_x\lambda_x & -\lambda_x\lambda_y & \lambda_x\lambda_x & \lambda_x\lambda_y \\ V_j & -\lambda_y\lambda_x & -\lambda_y\lambda_y & \lambda_y\lambda_x & \lambda_y\lambda_y \end{array}$$

Member	λ_x	λ_y
#1	-4/5 =-0.8	-3/5 = -0.6
#2	0	-3/3 = -1
#3	4/5 = 0.8	-3/5 = -0.6
#4	4/4 = 1	0
#5	4/4 = 1	0

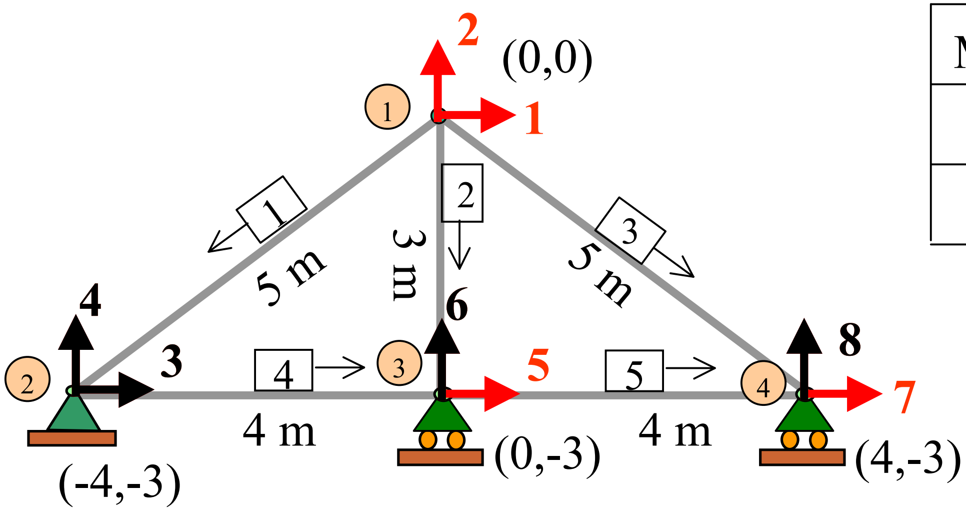


Member	λ_x	λ_y	λ_x^2/L	$\lambda_x\lambda_y/L$	λ_y^2/L
#1	-0.8	-0.6	0.128	0.096	0.072
#2	0	-1	0	0	0.333
#3	0.8	-0.6	0.128	-0.096	0.072

$$[k]_1 = 8\text{x}10^3 \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} 0.128 & 0.096 & -0.128 & -0.096 \\ 0.096 & 0.072 & -0.096 & -0.072 \\ -0.128 & -0.096 & 0.128 & 0.096 \\ -0.096 & -0.072 & 0.096 & 0.072 \end{pmatrix} \end{array}$$

$$[k]_3 = 8\text{x}10^3 \begin{array}{c} \\ 1 \\ 2 \\ 7 \\ 8 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 7 & 8 \end{array} \\ \begin{pmatrix} 0.128 & -0.096 & -0.128 & 0.096 \\ -0.096 & 0.072 & 0.096 & -0.072 \\ -0.128 & 0.096 & 0.128 & -0.096 \\ 0.096 & -0.072 & -0.096 & 0.072 \end{pmatrix} \end{array}$$

$$[k]_2 = 8\text{x}10^3 \begin{array}{c} \\ 1 \\ 2 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 5 & 6 \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.333 & 0 & -0.333 \\ 0 & 0 & 0 & 0 \\ 0 & -0.333 & 0 & 0.333 \end{pmatrix} \end{array}$$



Member	λ_x	λ_y	λ_x^2/L	$\lambda_x\lambda_y/L$	λ_y^2/L
#4	1	0	0.25	0	0
#5	1	0	0.25	0	0

$$[k]_4 = 8\text{x}10^3 \begin{array}{c} \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccc} 3 & 4 & 5 & 6 \end{array} \\ \begin{pmatrix} 0.25 & 0 & -0.25 & 0 \\ 0 & 0 & 0 & 0 \\ -0.25 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

$$[k]_5 = 8\text{x}10^3 \begin{array}{c} \\ 5 \\ 6 \\ 7 \\ 8 \end{array} \begin{array}{c} \begin{array}{cccc} 5 & 6 & 7 & 8 \end{array} \\ \begin{pmatrix} 0.25 & 0 & -0.25 & 0 \\ 0 & 0 & 0 & 0 \\ -0.25 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

Global Stiffness Matrix

$$[K] = 8\text{x}10^3 \begin{array}{c} \\ 1 \\ 2 \\ 5 \\ 7 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 5 & 7 \end{array} \\ \begin{pmatrix} & & & \\ & & & \\ & & & \\ & & & \end{pmatrix} \end{array}$$

Global Stiffness Matrix

$$[k]_1 = 8\text{x}10^3 \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0.128 & 0.096 & -0.128 & -0.096 \\ 2 & 0.096 & 0.072 & -0.096 & -0.072 \\ 3 & -0.128 & -0.096 & 0.128 & 0.096 \\ 4 & -0.096 & -0.072 & 0.096 & 0.072 \end{array}$$

$$[k]_4 = 8\text{x}10^3 \begin{array}{c|cccc} & 3 & 4 & 5 & 6 \\ \hline 3 & 0.25 & 0 & -0.25 & 0 \\ 4 & 0 & 0 & 0 & 0 \\ 5 & -0.25 & 0 & 0.25 & 0 \\ 6 & 0 & 0 & 0 & 0 \end{array}$$

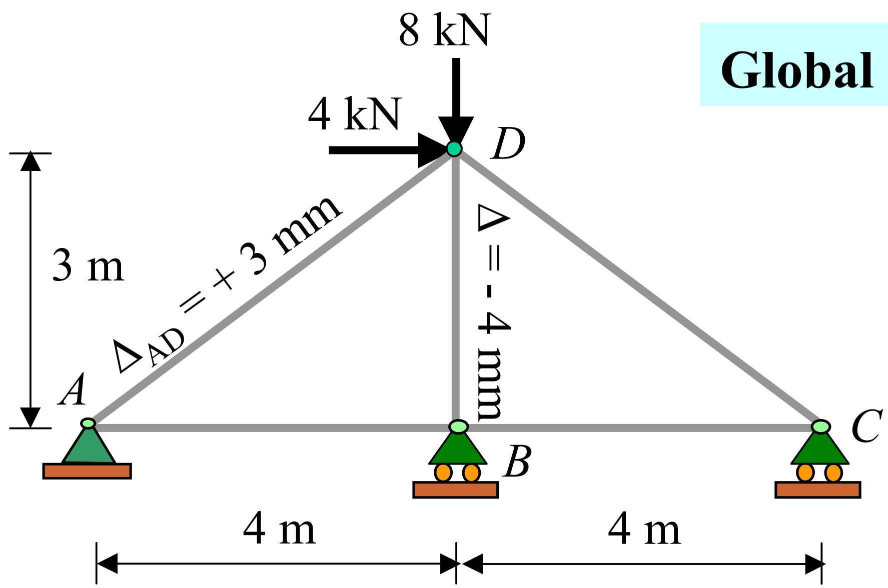
$$[k]_2 = 8\text{x}10^3 \begin{array}{c|cccc} & 1 & 2 & 5 & 6 \\ \hline 1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0.333 & 0 & -0.333 \\ 5 & 0 & 0 & 0 & 0 \\ 6 & 0 & -0.333 & 0 & 0.333 \end{array}$$

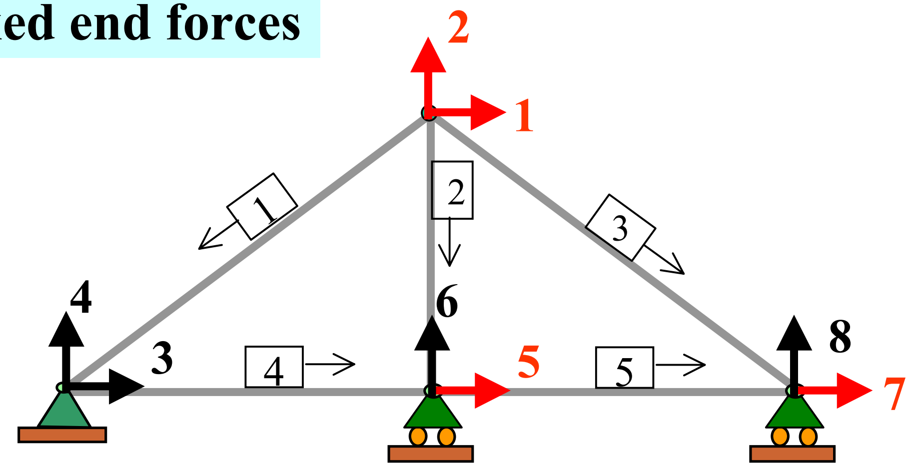
$$[k]_5 = 8\text{x}10^3 \begin{array}{c|cccc} & 5 & 6 & 7 & 8 \\ \hline 5 & 0.25 & 0 & -0.25 & 0 \\ 6 & 0 & 0 & 0 & 0 \\ 7 & -0.25 & 0 & 0.25 & 0 \\ 8 & 0 & 0 & 0 & 0 \end{array}$$

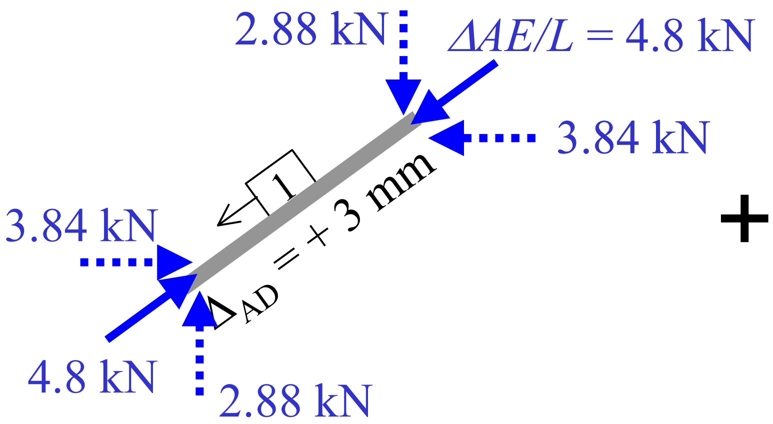
$$[k]_3 = 8\text{x}10^3 \begin{array}{c|cccc} & 1 & 2 & 7 & 8 \\ \hline 1 & 0.128 & -0.096 & -0.128 & 0.096 \\ 2 & -0.096 & 0.072 & 0.096 & -0.072 \\ 7 & -0.128 & 0.096 & 0.128 & -0.096 \\ 8 & 0.096 & -0.072 & -0.096 & 0.072 \end{array}$$

$$[K] = 8\text{x}10^3 \begin{array}{c|cccc} & 1 & 2 & 5 & 7 \\ \hline 1 & 0.256 & 0.0 & 0.0 & -0.128 \\ 2 & 0.0 & 0.477 & 0.0 & 0.096 \\ 5 & 0.0 & 0.0 & 0.50 & -0.25 \\ 7 & -0.128 & 0.096 & -0.25 & 0.378 \end{array}$$

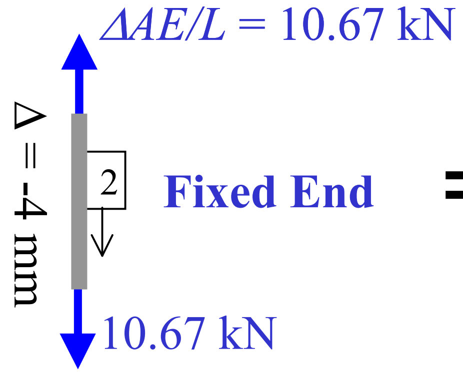
Global Fixed end forces



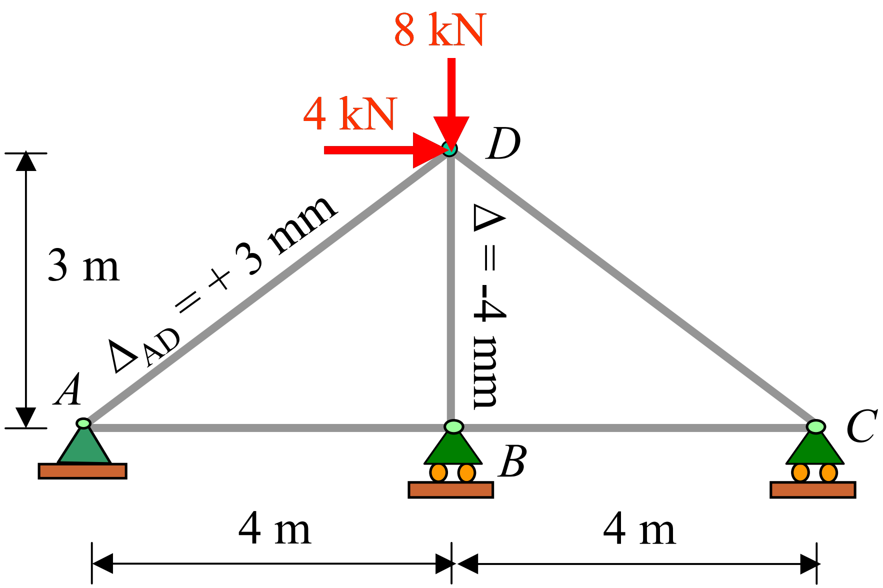


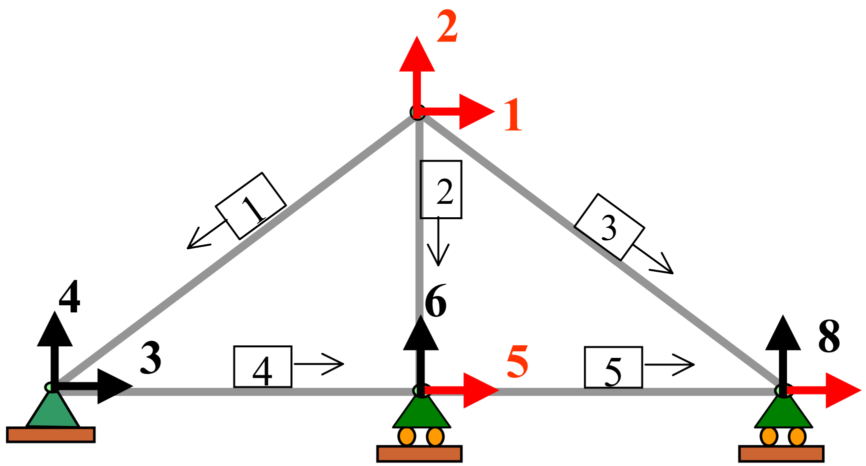


+



$$
=
\begin{array}{cc}
\begin{pmatrix}
-3.84 \\
-2.88 + 10.67 = 7.79 \\
0.0 \\
0.0
\end{pmatrix}
&
\begin{array}{c}
1 \\
2 \\
5 \\
7
\end{array}
\end{array}
$$

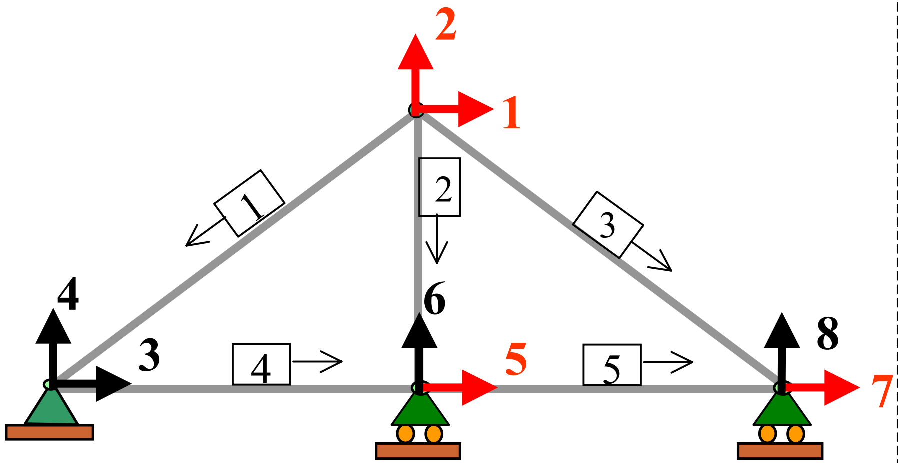




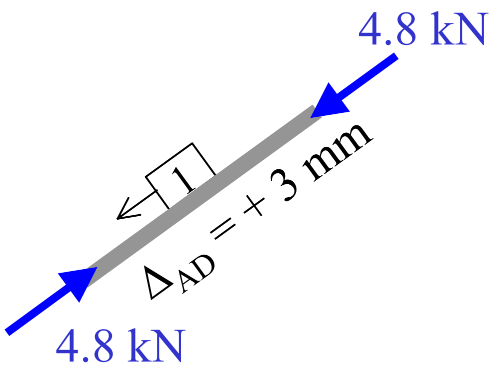
Global: $[Q] = [K][D] + [Q^F]$

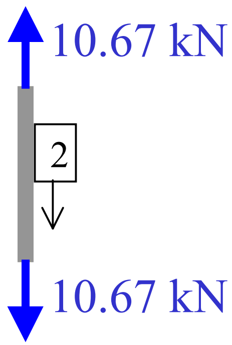
$$\begin{pmatrix} Q_1 = 4 \\ Q_2 = -8 \\ Q_5 = 0 \\ Q_7 = 0 \end{pmatrix} = 8\text{x}10^3 \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 5 & 7 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 5 \\ 7 \end{array} & \begin{pmatrix} 0.256 & 0.0 & 0.0 & -0.128 \\ 0.0 & 0.477 & 0.0 & 0.096 \\ 0.0 & 0.0 & 0.50 & -0.25 \\ -0.128 & 0.096 & -0.25 & 0.378 \end{pmatrix} \end{array} \begin{pmatrix} D_1 \\ D_2 \\ D_5 \\ D_7 \end{pmatrix} + \begin{pmatrix} -3.84 \\ 7.79 \\ 0.0 \\ 0.0 \end{pmatrix}$$

$$\begin{pmatrix} D_1 \\ D_2 \\ D_5 \\ D_7 \end{pmatrix} = \begin{pmatrix} 6.4426\text{x}10^{-3} & \text{m} \\ -5.1902\text{x}10^{-3} & \text{m} \\ 2.6144\text{x}10^{-3} & \text{m} \\ 5.2288\text{x}10^{-3} & \text{m} \end{pmatrix}$$



$$\begin{Bmatrix} D_1 \\ D_2 \\ D_5 \\ D_7 \end{Bmatrix} = \begin{Bmatrix} 6.4426\text{x}10^{-3} & \text{m} \\ -5.1902\text{x}10^{-3} & \text{m} \\ 2.6144\text{x}10^{-3} & \text{m} \\ 5.2288\text{x}10^{-3} & \text{m} \end{Bmatrix}$$





Member forces

$$[q'_F]_m = \frac{AE}{L}\begin{bmatrix} -\lambda_x & -\lambda_y & \lambda_x & \lambda_x \end{bmatrix}\begin{bmatrix} D_{xi} \\ D_{yi} \\ D_{xj} \\ D_{yj} \end{bmatrix} + \begin{bmatrix} q'^F \end{bmatrix}$$

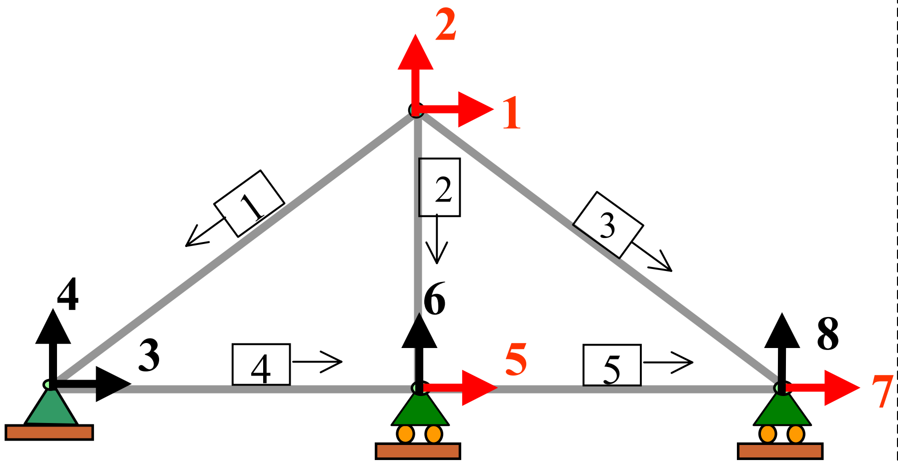
Member	λ_{ix}	λ_{iy}
#1	-0.8	-0.6
#2	0	-1

$$[q'_F]_1 = \frac{8\text{x}10^3}{5}\begin{bmatrix} 0.8 & 0.6 & -0.8 & -0.6 \end{bmatrix}\begin{Bmatrix} D_1 \\ D_2 \\ 0 \\ 0 \end{Bmatrix} + \begin{bmatrix} -4.8 \end{bmatrix}$$

$$= -1.54 \text{ kN (C)}$$

$$[q'_F]_2 = \frac{8\text{x}10^3}{3}\begin{bmatrix} 0.0 & 1.0 & 0.0 & -1.0 \end{bmatrix}\begin{Bmatrix} D_1 \\ D_2 \\ D_5 \\ 0 \end{Bmatrix} + \begin{bmatrix} 10.67 \end{bmatrix}$$

$$= -3.17 \text{ kN (C)}$$



$$\begin{pmatrix} D_1 \\ D_2 \\ D_5 \\ D_7 \end{pmatrix} = \begin{pmatrix} 6.4426\text{x}10^{-3} & \text{m} \\ -5.1902\text{x}10^{-3} & \text{m} \\ 2.6144\text{x}10^{-3} & \text{m} \\ 5.2288\text{x}10^{-3} & \text{m} \end{pmatrix}$$

Member	λ_x	λ_y
#3	0.8	-0.6
#4	1	0
#5	1	0

$$[q'_F]_m = \frac{AE}{L}\begin{bmatrix} -\lambda_x & -\lambda_y & \lambda_x & \lambda_x \end{bmatrix}\begin{bmatrix} D_{xi} \\ D_{yi} \\ D_{xj} \\ D_{yj} \end{bmatrix} + \begin{bmatrix} q'^F \end{bmatrix}$$

$$[q'_F]_3 = \frac{8\text{x}10^3}{5}\begin{bmatrix} -0.8 & 0.6 & 0.8 & -0.6 \end{bmatrix}\begin{pmatrix} D_1 \\ D_2 \\ D_7 \\ 0 \end{pmatrix}$$

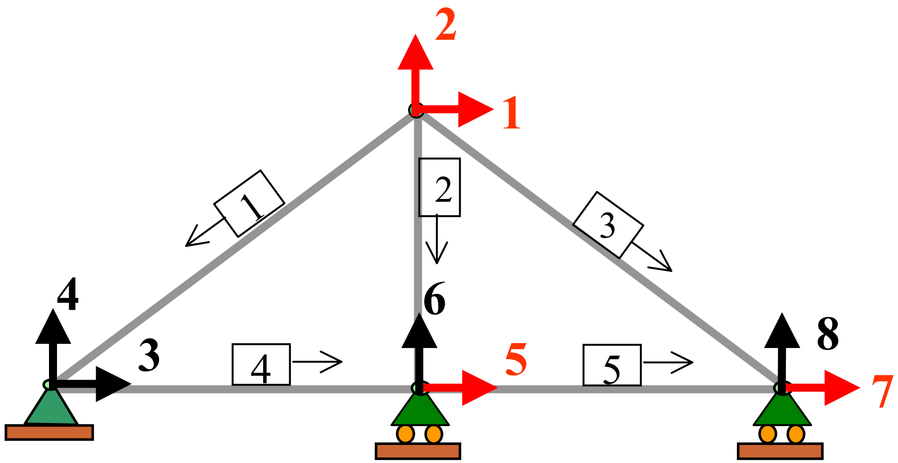
$$= -6.54 \text{ kN (C)}$$

$$[q'_F]_4 = \frac{8\text{x}10^3}{4}\begin{bmatrix} -1.0 & 0.0 & 1.0 & 0.0 \end{bmatrix}\begin{pmatrix} 0 \\ 0 \\ D_5 \\ 0 \end{pmatrix}$$

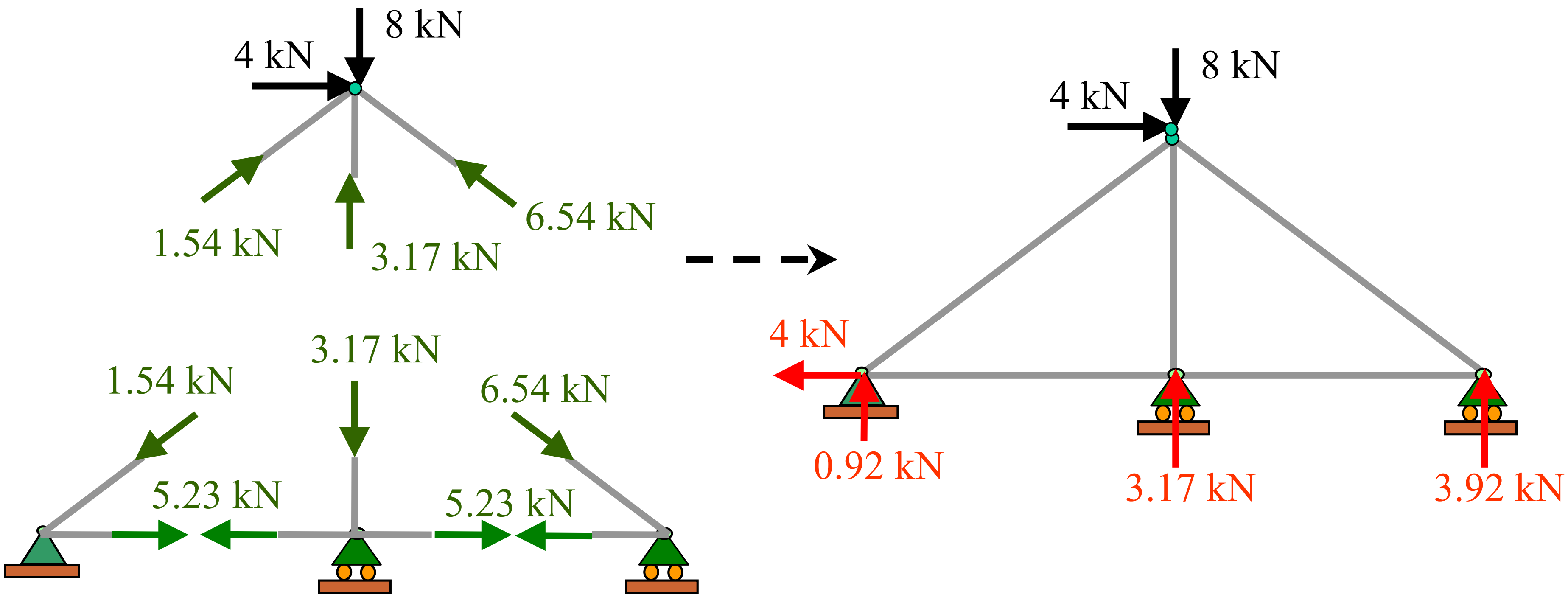
$$= 5.23 \text{ kN (T)}$$

$$[q'_F]_5 = \frac{8\text{x}10^3}{4}\begin{bmatrix} -1.0 & 0.0 & 1.0 & 0.0 \end{bmatrix}\begin{pmatrix} D_5 \\ 0 \\ D_7 \\ 0 \end{pmatrix}$$

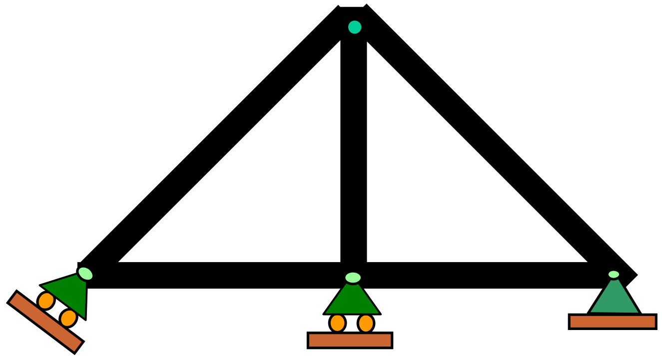
$$= 5.23 \text{ kN (T)}$$



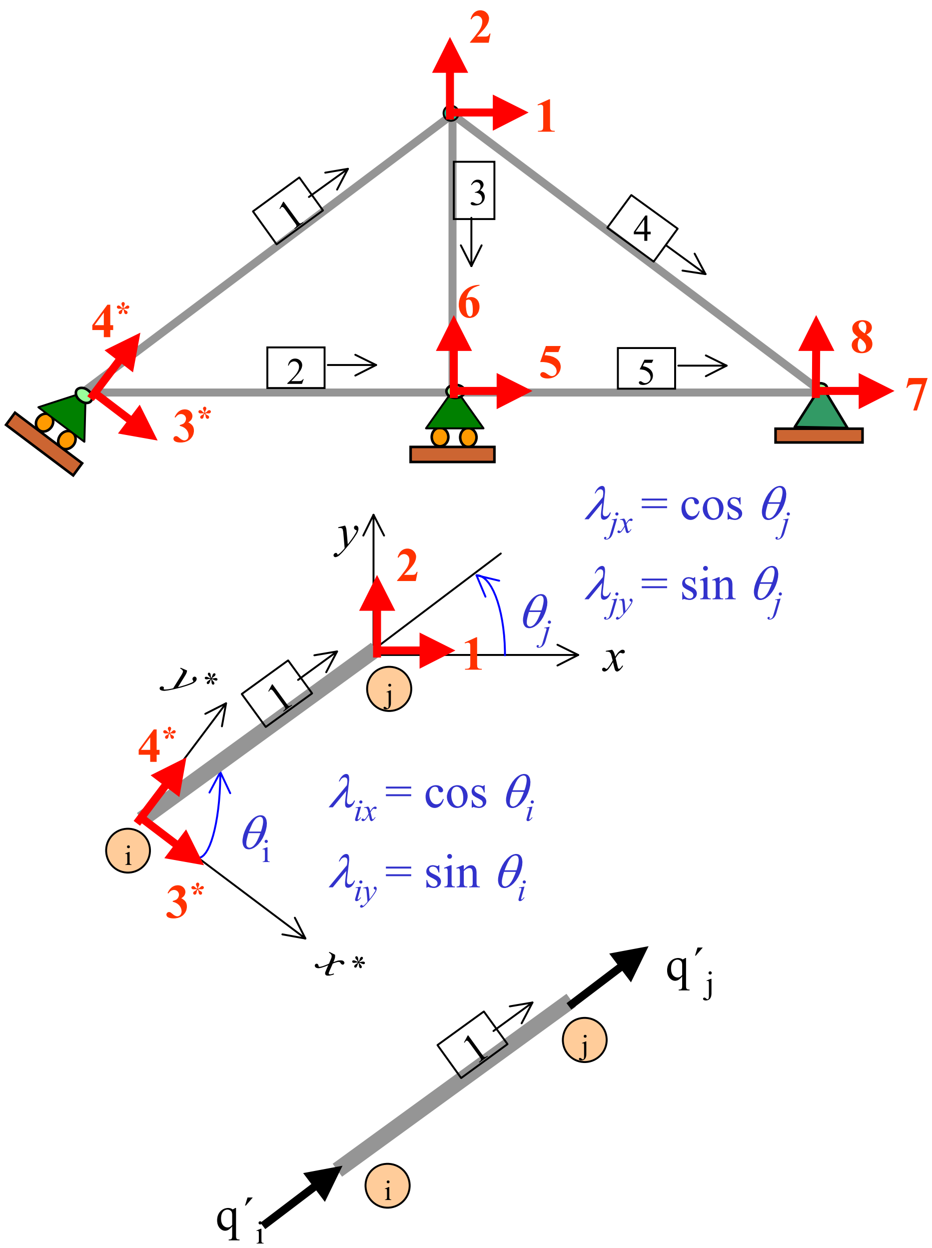
Member	λ_x	λ_y	$[q']$
#1	-0.8	-0.6	-1.54
#2	0	-1	-3.17
#3	0.8	-0.6	-6.54
#4	1	0	5.23
#5	1	0	5.23



Special Trusses (Inclined roller supports)



▶ Transformation Matrices



$$\lambda_{jx} = \cos\,\theta_j$$

$$\lambda_{jy} = \sin\,\theta_j$$

$$\lambda_{ix} = \cos\,\theta_i$$

$$\lambda_{iy} = \sin\,\theta_i$$

$$[\,\mathbf{q}^*\,] = [\,\mathbf{T}\,]^{\mathrm{T}}[\,\mathbf{q}'\,]$$

$$\begin{pmatrix} q^*{}_3 \\ q^*{}_4 \\ q_1 \\ q_2 \end{pmatrix} = \underbrace{\begin{pmatrix} \lambda_{ix} & 0 \\ \lambda_{iy} & 0 \\ 0 & \lambda_{jx} \\ 0 & \lambda_{jy} \end{pmatrix}}_{[\mathrm{T}]^{\mathrm{T}}} \begin{pmatrix} q'{}_{\mathrm{i}} \\ q'{}_{\mathrm{j}} \end{pmatrix}$$

$$[\,\mathbf{T}\,] = [[\,\mathbf{T}\,]^{\mathrm{T}}]^{\mathrm{T}} = \begin{bmatrix} \lambda_{ix} & \lambda_{iy} & 0 & 0 \\ 0 & 0 & \lambda_{jx} & \lambda_{jy} \end{bmatrix}$$

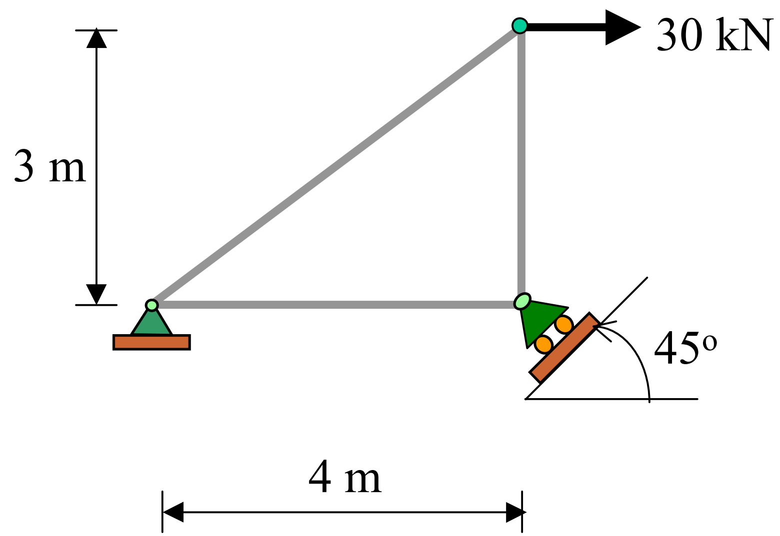
[k] = [T]^{T}[k′][T]

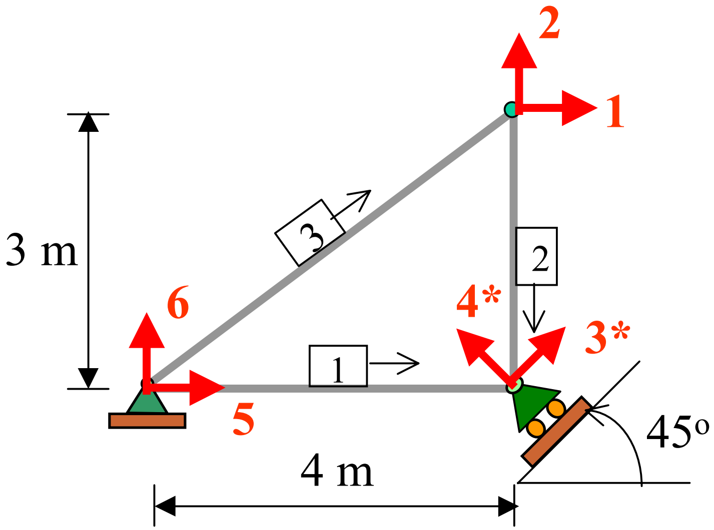
$$[\,k\,]_{\mathrm{m}} = \begin{pmatrix} \lambda_{ix} & 0 \\ \lambda_{iy} & 0 \\ 0 & \lambda_{jx} \\ 0 & \lambda_{jy} \end{pmatrix} \frac{AE}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \lambda_{ix} & \lambda_{iy} & 0 & 0 \\ 0 & 0 & \lambda_{jx} & \lambda_{jy} \end{bmatrix}$$

$$[\,k\,]_{\mathrm{m}} = \frac{AE}{L} \begin{array}{c} \\ U_i \\ V_i \\ U_j \\ V_j \end{array} \begin{array}{c} \begin{array}{cccc} U_i & V_i & U_j & V_j \end{array} \\ \begin{pmatrix} \lambda_{ix}\lambda_{ix} & \lambda_{ix}\lambda_{iy} & -\lambda_{ix}\lambda_{jx} & -\lambda_{ix}\lambda_{jy} \\ \lambda_{iy}\lambda_{ix} & \lambda_{iy}\lambda_{iy} & -\lambda_{iy}\lambda_{jx} & -\lambda_{iy}\lambda_{jy} \\ -\lambda_{jx}\lambda_{ix} & -\lambda_{jx}\lambda_{iy} & \lambda_{jx}\lambda_{jx} & \lambda_{jx}\lambda_{jy} \\ -\lambda_{jy}\lambda_{ix} & -\lambda_{jy}\lambda_{iy} & \lambda_{jy}\lambda_{jx} & \lambda_{jy}\lambda_{jy} \end{pmatrix} \end{array}$$

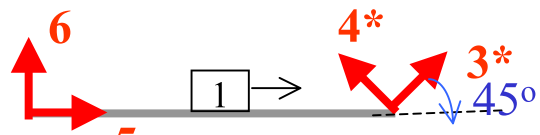
Example 5

For the truss shown, use the stiffness method to:
(b) Determine the **end forces** of each member and **reactions** at supports.
(a) Determine the **displacement** of the loaded joint.
AE is constant.





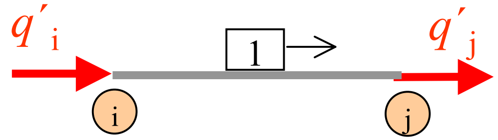
Member 1:



$\theta_i = 0$,
$\lambda_{ix} = \cos 0 = 1$,
$\lambda_{iy} = \sin 0 = 0$

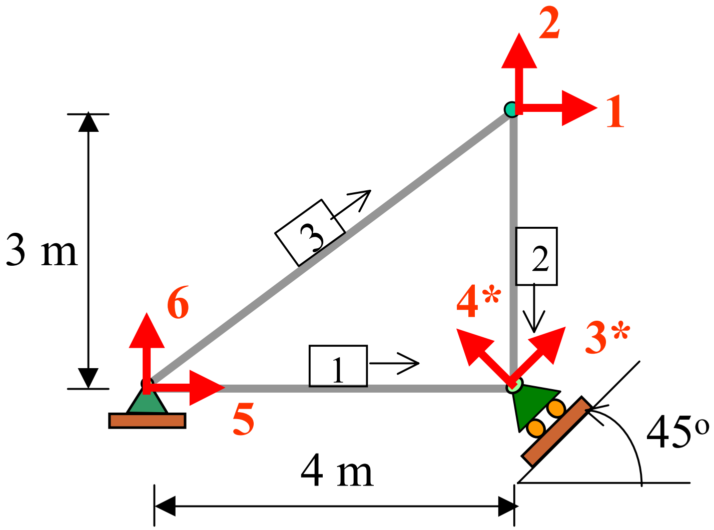
[q*]

$\theta_{ij} = -45^{o} = 135^{o}$,
$\lambda_{ix} = \cos(-45^{o}) = 0.707$,
$\lambda_{iy} = \sin(-45^{o}) = -0.707$



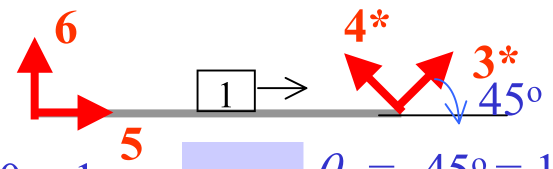
$$[q^*] = [T^*]^T[q'] + [T^*]^T[q'^F]$$

$$\begin{bmatrix} q_5 \\ q_6 \\ q_{3*} \\ q_{4*} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0.707 \\ 0 & -0.707 \end{bmatrix}}_{[T^*]^T} \begin{bmatrix} q'_i \\ q'_j \end{bmatrix}$$



$$[k^*]=[T^T]\frac{AE}{L}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}[T]$$

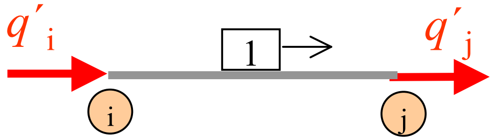
Member 1:



$\theta_i = 0$;
$\lambda_{ix} = \cos 0 = 1$,
$\lambda_{iy} = \sin 0 = 0$

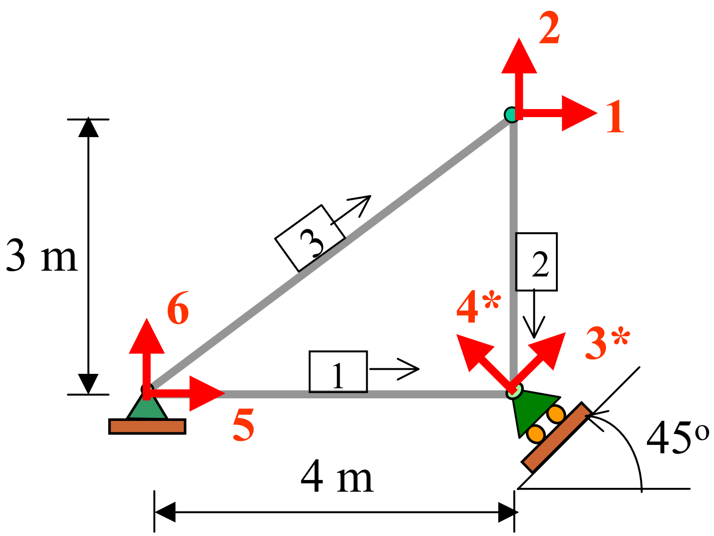
[q*]

θ_{ij} = -45° = 135°,
$\lambda_{ix} = \cos(-45^\circ) = 0.707$,
$\lambda_{iy} = \sin(-45^\circ) = -0.707$



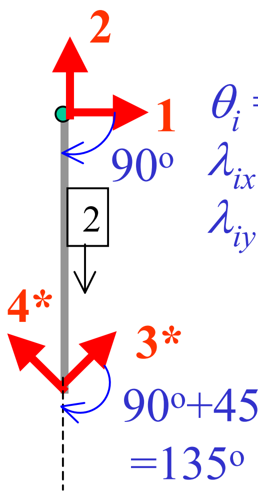
$$[k^*]_1 = \begin{pmatrix}1 & 0\\ 0 & 0\\ 0 & 0.707\\ 0 & -0.707\end{pmatrix}\frac{AE}{4}\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 0.707 & -0.707\end{pmatrix}$$

$$[k^*]_1 = AE \begin{array}{c} \\ 5 \\ 6 \\ 3^* \\ 4^* \end{array}\begin{array}{c} \begin{array}{cccc} 5 & 6 & 3^* & 4^* \end{array} \\ \begin{pmatrix} 0.25 & 0 & -0.1768 & 0.1768\\ 0 & 0 & 0 & 0\\ -0.1768 & 0 & 0.125 & -0.125\\ 0.1768 & 0 & -0.125 & 0.125\end{pmatrix}\end{array}$$



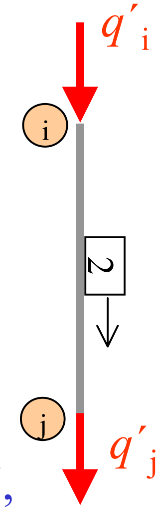
$$[k^*] = [T^T]\frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}[T]$$

Member 2:



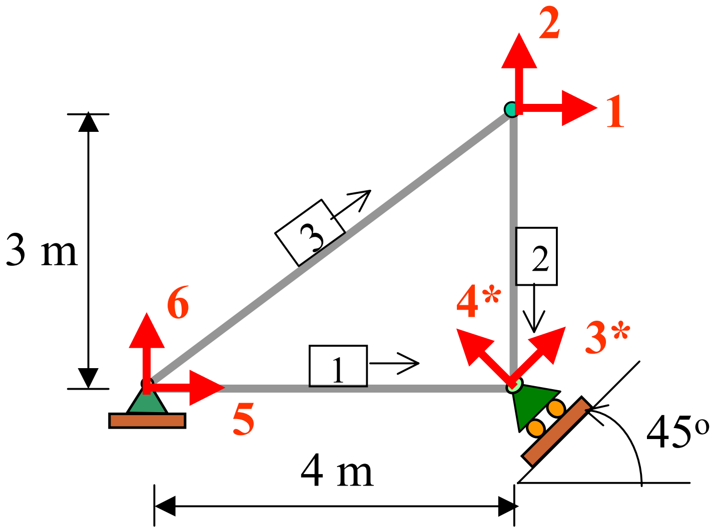
$\theta_i = -90^o = 270^o$,
$\lambda_{ix} = \cos(-90^o) = 0$,
$\lambda_{iy} = \sin(-90^o) = -1$

$\theta_j = -135^o = 215^o$,
$\lambda_{ix} = \cos(-135^o) = -0.707$,
$\lambda_{iy} = \sin(-135^o) = -0.707$

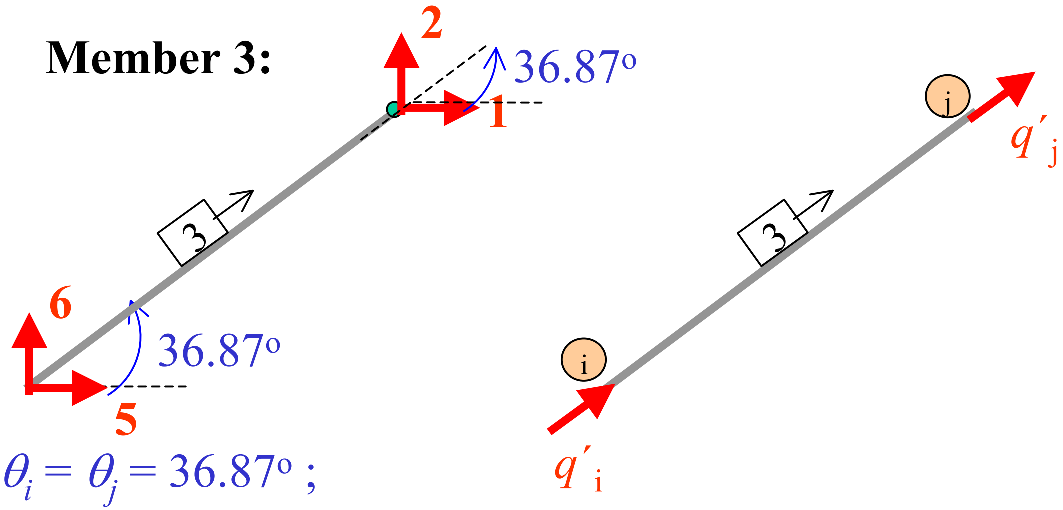


$$[k^*]_2 = \begin{pmatrix} 0 & 0 \\ -1 & 0 \\ 0 & -0.707 \\ 0 & -0.707 \end{pmatrix} \frac{AE}{3} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -0.707 & -0.707 \end{pmatrix}$$

$$[k^*]_2 = AE \begin{array}{c} \\ 1 \\ 2 \\ 3^* \\ 4^* \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3^* & 4^* \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.3333 & -0.2357 & -0.2357 \\ 0 & -0.2357 & 0.1667 & 0.1667 \\ 0 & -0.2357 & 0.1667 & 0.1667 \end{pmatrix} \end{array}$$



Member 3:



$$[k] = [T^T]\frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}[T]$$

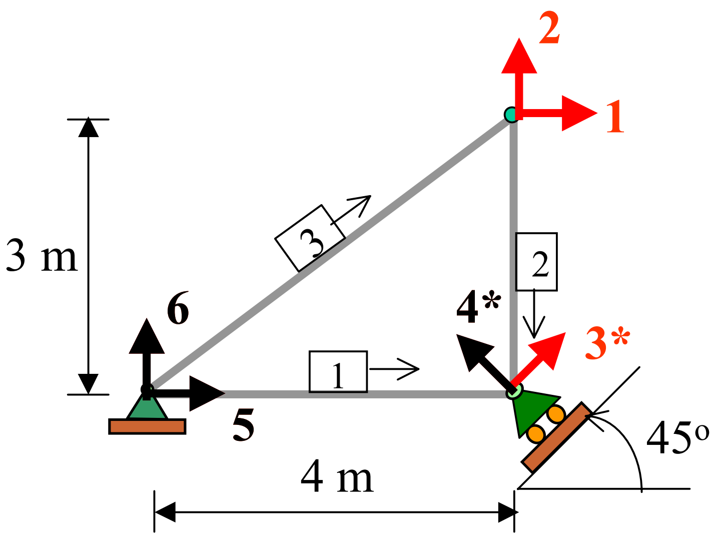
$\theta_i = \theta_j = 36.87^{o}$;

$\lambda_{ix} = \lambda_{jx} = \cos(36.87^{o}) = 0.8,$

$\lambda_{iy} = \lambda_{jy} = \sin(36.87^{o}) = 0.6$

$$[k]_3 = \begin{pmatrix} 0.8 & 0 \\ 0.6 & 0 \\ 0 & 0.8 \\ 0 & 0.6 \end{pmatrix} \frac{AE}{5} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 0.8 & 0.6 & 0 & 0 \\ 0 & 0 & 0.8 & 0.6 \end{pmatrix}$$

$$[k]_3 = AE \begin{array}{c} \\ 5 \\ 6 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{cccc} 5 & 6 & 1 & 2 \end{array} \\ \begin{pmatrix} 0.128 & 0.096 & -0.128 & -0.096 \\ 0.096 & 0.072 & -0.096 & -0.072 \\ -0.128 & -0.096 & 0.128 & 0.096 \\ -0.096 & -0.072 & 0.096 & 0.072 \end{pmatrix} \end{array}$$



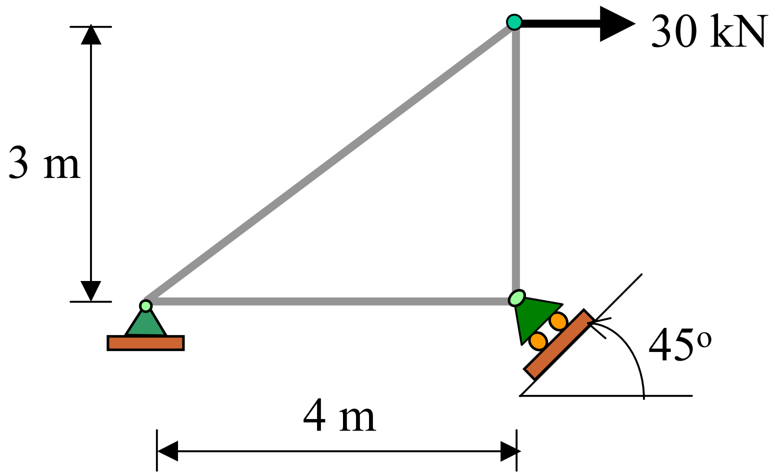
Global Stiffness:

$$[k^*]_1 = AE \begin{array}{c} \\ 5 \\ 6 \\ 3^* \\ 4^* \end{array} \begin{array}{c} \begin{array}{cccc} 5 & 6 & 3^* & 4^* \end{array} \\ \begin{pmatrix} 0.25 & 0 & -0.1768 & 0.1768 \\ 0 & 0 & 0 & 0 \\ -0.1768 & 0 & 0.125 & -0.125 \\ 0.1768 & 0 & -0.125 & 0.125 \end{pmatrix} \end{array}$$

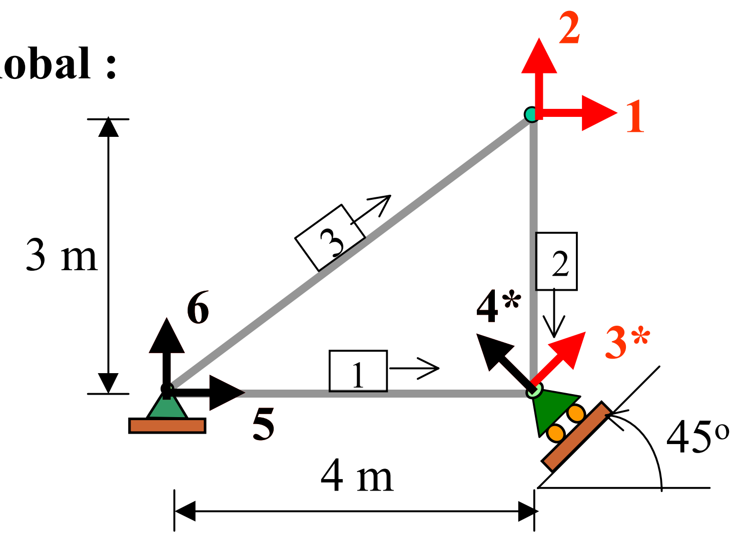
$$[k^*]_2 = AE \begin{array}{c} \\ 1 \\ 2 \\ 3^* \\ 4^* \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3^* & 4^* \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.3333 & -0.2357 & -0.2357 \\ 0 & -0.2357 & 0.1667 & 0.1667 \\ 0 & -0.2357 & 0.1667 & 0.1667 \end{pmatrix} \end{array}$$

$$[K] = AE \begin{array}{c} \\ 1 \\ 2 \\ 3^* \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3^* \end{array} \\ \begin{pmatrix} 0.128 & 0.096 & 0 \\ 0.096 & 0.4053 & -0.2357 \\ 0 & -0.2357 & 0.2917 \end{pmatrix} \end{array}$$

$$[k]_3 = AE \begin{array}{c} \\ 5 \\ 6 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{cccc} 5 & 6 & 1 & 2 \end{array} \\ \begin{pmatrix} 0.128 & 0.096 & -0.128 & -0.096 \\ 0.096 & 0.072 & -0.096 & -0.072 \\ -0.128 & -0.096 & 0.128 & 0.096 \\ -0.096 & -0.072 & 0.096 & 0.072 \end{pmatrix} \end{array}$$



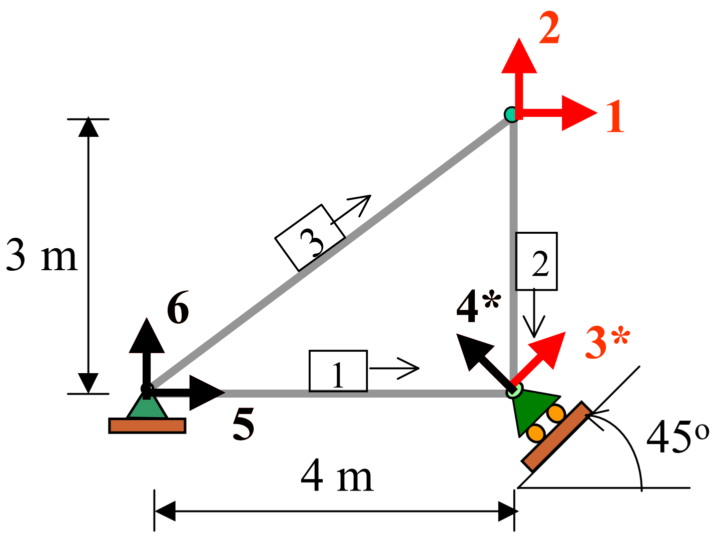
Global :



$[Q] = [K][D] + [Q^F]$

$$\begin{pmatrix} Q_1 = 30 \\ Q_2 = 0 \\ Q_{3*} = 0 \end{pmatrix} = AE \begin{array}{c} \\ 1 \\ 2 \\ 3* \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3* \end{array} \\ \begin{pmatrix} 0.128 & 0.096 & 0 \\ 0.096 & 0.4053 & -0.2357 \\ 0 & -0.2357 & 0.2917 \end{pmatrix} \end{array} \begin{pmatrix} D_1 \\ D_2 \\ D_{3*} \end{pmatrix}$$

$$\begin{pmatrix} D_1 \\ D_2 \\ D_{3*} \end{pmatrix} = \frac{1}{AE} \begin{pmatrix} 352.5 \\ -157.5 \\ -127.3 \end{pmatrix}$$



$$\begin{pmatrix} D_1 \\ D_2 \\ D_{3*} \end{pmatrix} = \frac{1}{AE} \begin{pmatrix} 352.5 \\ -157.5 \\ -127.3 \end{pmatrix}$$

Member	λ_{ix}	λ_{iy}	λ_{jx}	λ_{jy}
#1	1	0	0.707	-0.707
#2	0	-1	-0.707	-0.707
#3	0.8	0.6	0.8	0.6

Member Forces :

$$[q'_F]_m = \frac{AE}{L}\begin{bmatrix} -\lambda_x & -\lambda_y & \lambda_x & \lambda_x \end{bmatrix} \begin{bmatrix} D_{xi} \\ D_{yi} \\ D_{xj} \\ D_{yj} \end{bmatrix} + \left[q'^F\right]$$

$$[q'_F]_1 = \frac{AE}{4}\begin{bmatrix} -1 & 0 & 0.707 & -0.707 \end{bmatrix} \begin{pmatrix} 0 \\ 0 \\ D_{3*} \\ 0 \end{pmatrix}$$

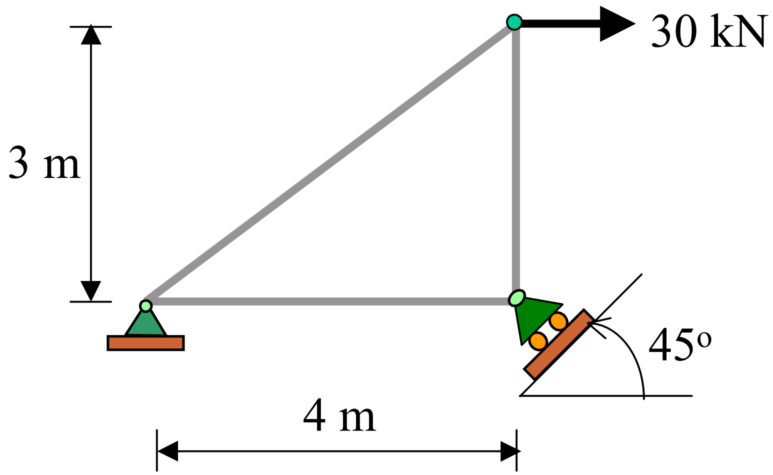
$= -22.50$ kN, (C)

$$[q'_F]_2 = \frac{AE}{3}\begin{bmatrix} 0 & 1 & -0.707 & -0.707 \end{bmatrix} \begin{pmatrix} D_1 \\ D_2 \\ D_{3*} \\ 0 \end{pmatrix}$$

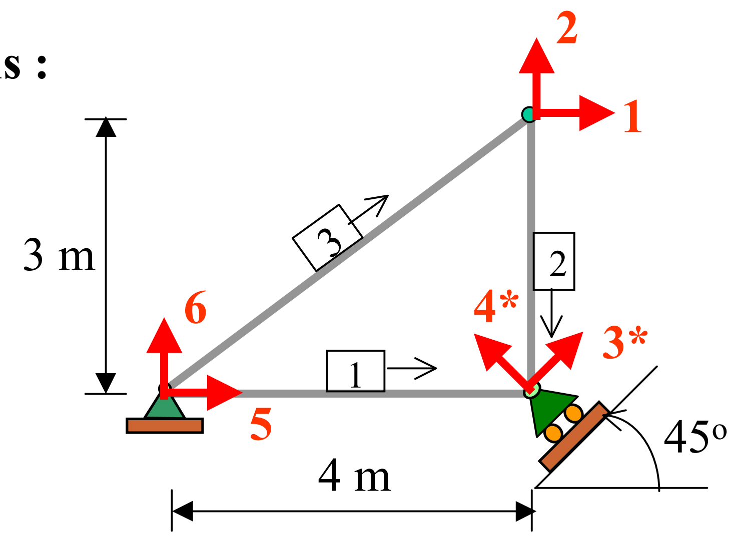
$= -22.50$ kN, (C)

$$[q'_F]_3 = \frac{AE}{5}\begin{bmatrix} -0.8 & -0.6 & 0.8 & 0.6 \end{bmatrix} \begin{pmatrix} 0 \\ 0 \\ D_1 \\ D_2 \end{pmatrix}$$

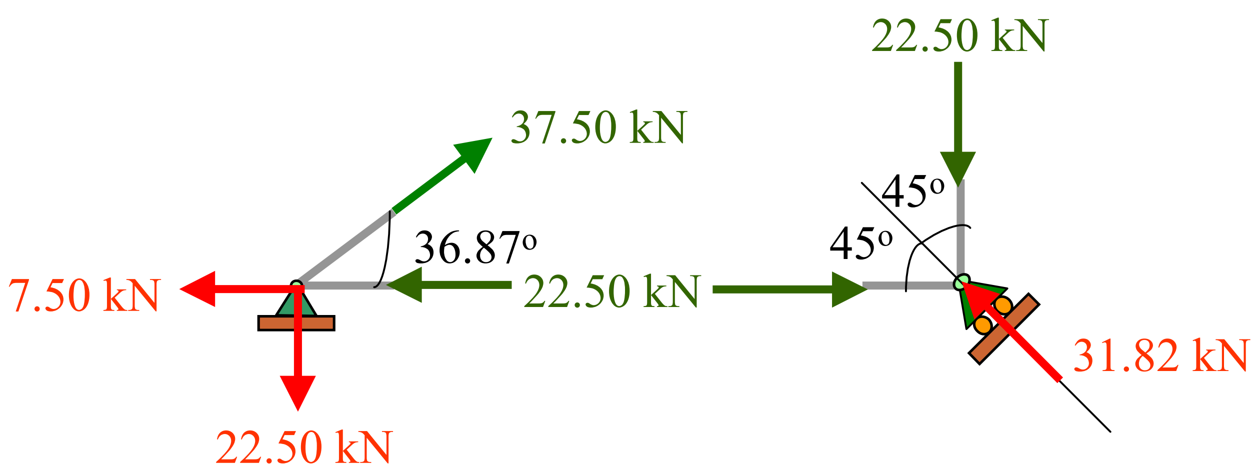
$= 37.50$ kN, (T)



Reactions :

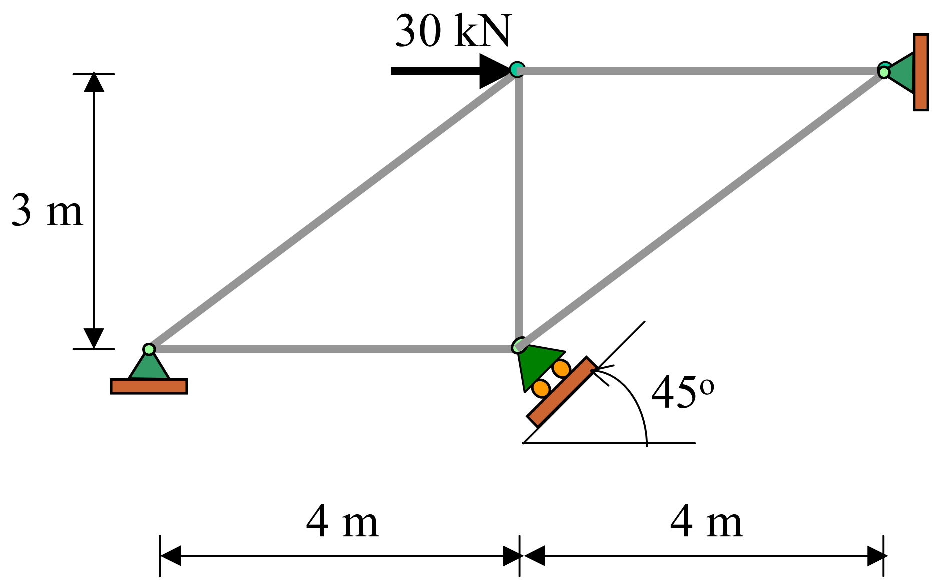


Member	$[q']_1$	$[q']_2$	$[q']_3$
Member Force (kN)	-22.50	-22.50	37.50

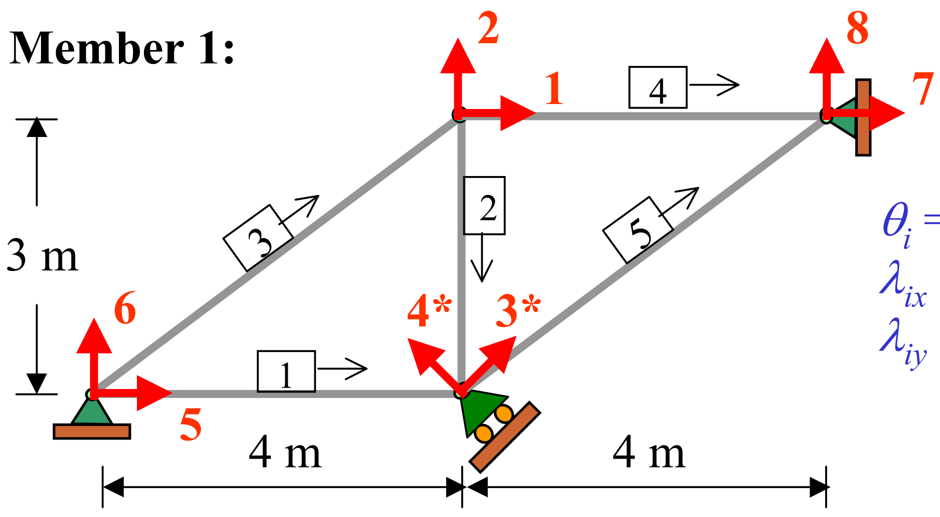


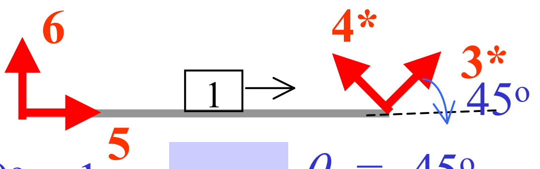
Example 6

For the truss shown, use the stiffness method to:
(b) Determine the **end forces** of each member and **reactions** at supports.
(a) Determine the **displacement** of the loaded joint.
AE is constant.



Member 1:

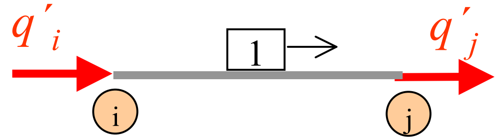




$\theta_i = 0°$,
$\lambda_{ix} = \cos 0° = 1$,
$\lambda_{iy} = \sin 0° = 0$

[q*]

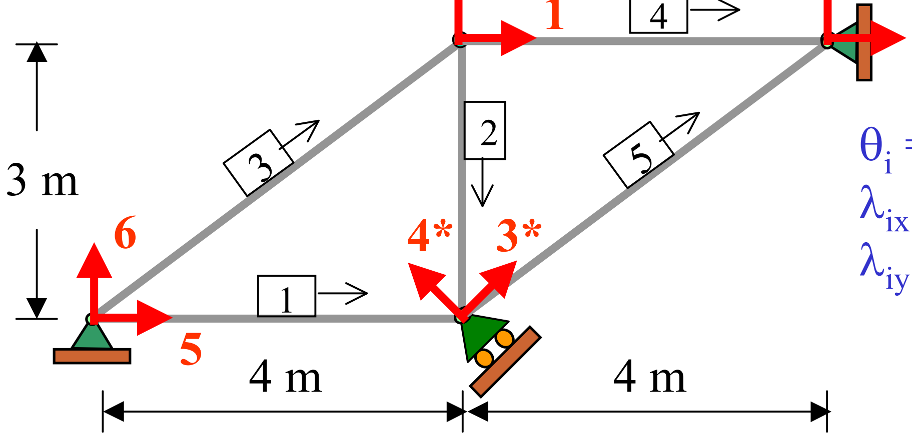
$\theta_{ij} = -45°$
$\lambda_{ix} = \cos(-45°) = 0.707$,
$\lambda_{iy} = \sin(-45°) = -0.707$

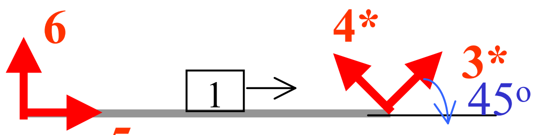


$$[q^*] = [T^*]^T[q'] + [T^*]^T[q'^F]$$

$$\begin{bmatrix} q_5 \\ q_6 \\ q_{3*} \\ q_{4*} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0.707 \\ 0 & -0.707 \end{bmatrix}}_{[T^*]^T} \begin{bmatrix} q'_i \\ q'_j \end{bmatrix}$$

Member 1:

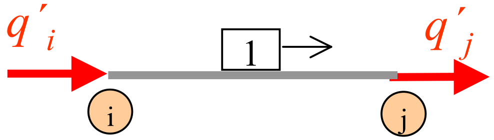




$\theta_i = 0^o,$
$\lambda_{ix} = \cos 0^o = 1,$
$\lambda_{iy} = \sin 0^o = 0$

[q*]

$\theta_{ij} = -45^o = 315^o,$
$\lambda_{ix} = \cos(-45^o) = 0.707,$
$\lambda_{iy} = \sin(-45^o) = -0.707$

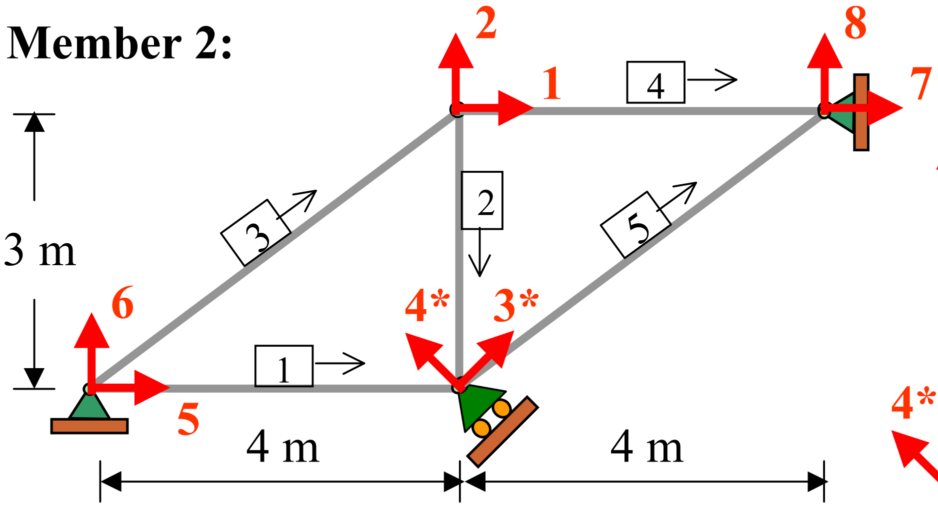


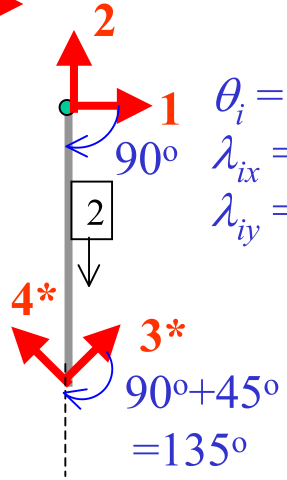
$$[k^*] = [T^T]\frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}[T]$$

$$[k^*]_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0.707 \\ 0 & -0.707 \end{pmatrix} \frac{AE}{4} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0.707 & -0.707 \end{pmatrix}$$

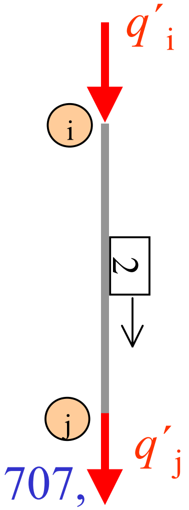
$$[k^*]_1 = AE \begin{array}{c} \\ 5 \\ 6 \\ 3^* \\ 4^* \end{array} \begin{array}{c} \begin{array}{cccc} 5 & 6 & 3^* & 4^* \end{array} \\ \begin{pmatrix} 0.25 & 0 & -0.1768 & 0.1768 \\ 0 & 0 & 0 & 0 \\ -0.1768 & 0 & 0.125 & -0.125 \\ 0.1768 & 0 & -0.125 & 0.125 \end{pmatrix} \end{array}$$

Member 2:





θ_i = -90°,
λ_{ix} = cos(-90°) = 0,
λ_{iy} = sin(-90°) = -1



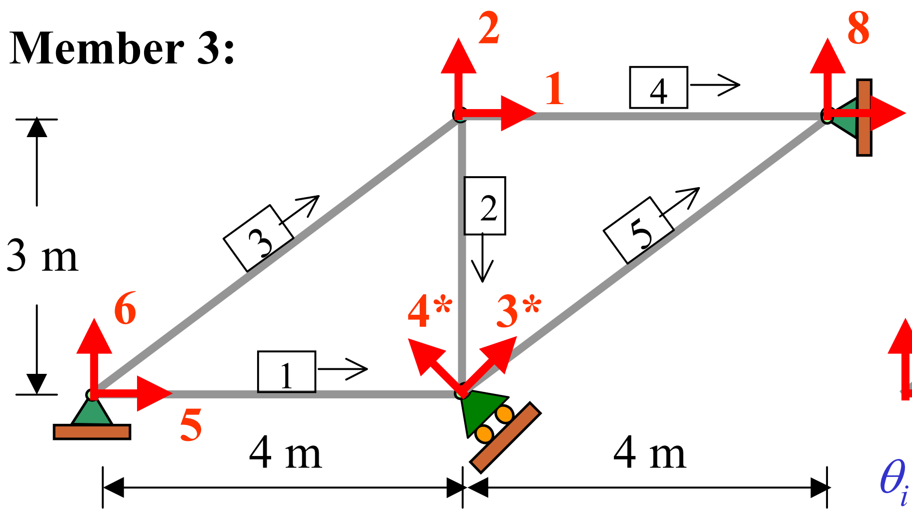
θ_j = -135° = 215°,
λ_{ix} = cos (-135°) = -0.707,
λ_{iy} = sin(- 135°) = -0.707

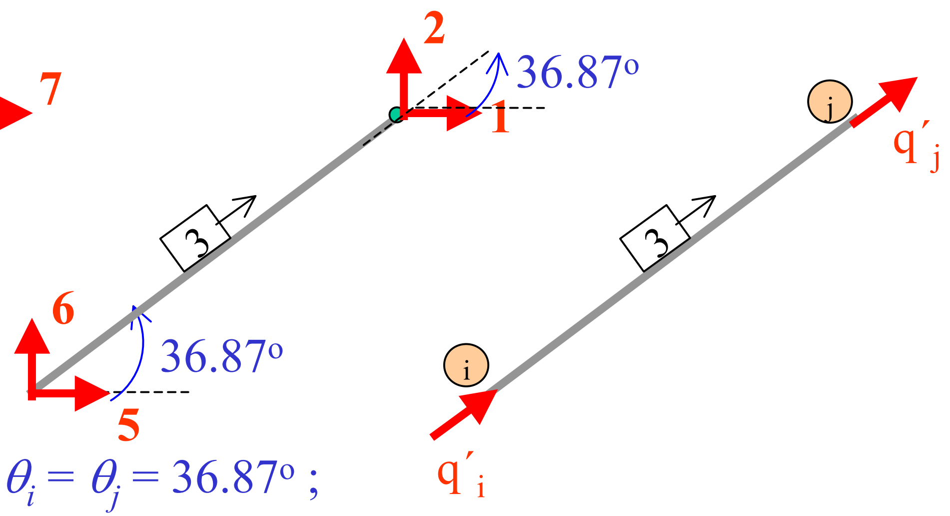
$$[k^*] = [T^T]\frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}[T]$$

$$[k^*]_2 = \begin{pmatrix} 0 & 0 \\ -1 & 0 \\ 0 & -0.707 \\ 0 & -0.707 \end{pmatrix} \frac{AE}{3} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -0.707 & -0.707 \end{bmatrix}$$

$$[k^*]_2 = AE \begin{array}{c} \\ 1 \\ 2 \\ 3^* \\ 4^* \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3^* & 4^* \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.3333 & -0.2357 & -0.2357 \\ 0 & -0.2357 & 0.1667 & 0.1667 \\ 0 & -0.2357 & 0.1667 & 0.1667 \end{pmatrix} \end{array}$$

Member 3:





$\theta_i = \theta_j = 36.87^o$;

$\lambda_{ix} = \lambda_{jx} = \cos(36.87^o) = 0.8,$

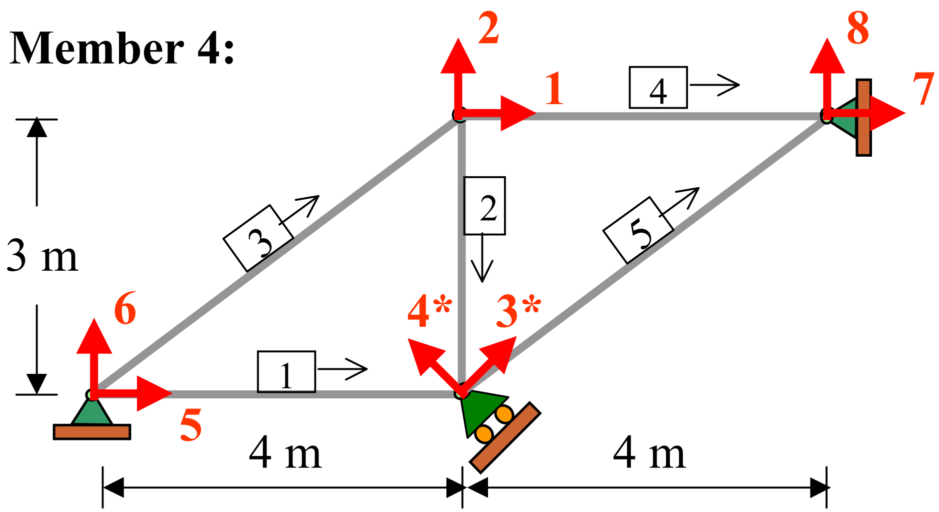
$\lambda_{iy} = \lambda_{jy} = \sin(36.87^o) = 0.6$

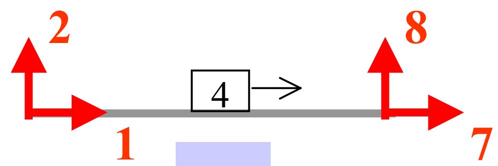
$$\boxed{[k^*] = [T^T]\frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}[T]}$$

$$[\mathrm{k}]_3 = \begin{pmatrix} 0.8 & 0 \\ 0.6 & 0 \\ 0 & 0.8 \\ 0 & 0.6 \end{pmatrix} \frac{AE}{5} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 0.8 & 0.6 & 0 & 0 \\ 0 & 0 & 0.8 & 0.6 \end{pmatrix}$$

$$[\mathrm{k}]_3 = AE \begin{array}{c} \\ 5 \\ 6 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{cccc} 5 & 6 & 1 & 2 \end{array} \\ \begin{pmatrix} 0.128 & 0.096 & -0.128 & -0.096 \\ 0.096 & 0.072 & -0.096 & -0.072 \\ -0.128 & -0.096 & 0.128 & 0.096 \\ -0.096 & -0.072 & 0.096 & 0.072 \end{pmatrix} \end{array}$$

Member 4:

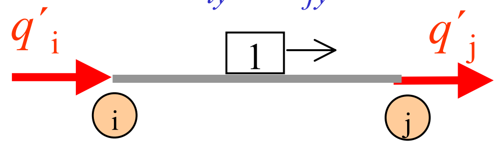




[q]

$\theta_i = \theta_{ij} = 0^o$; $\lambda_{ix} = \lambda_{jx} = \cos 0^o = 1,$

$\lambda_{iy} = \lambda_{jy} = \sin 0^o = 0$

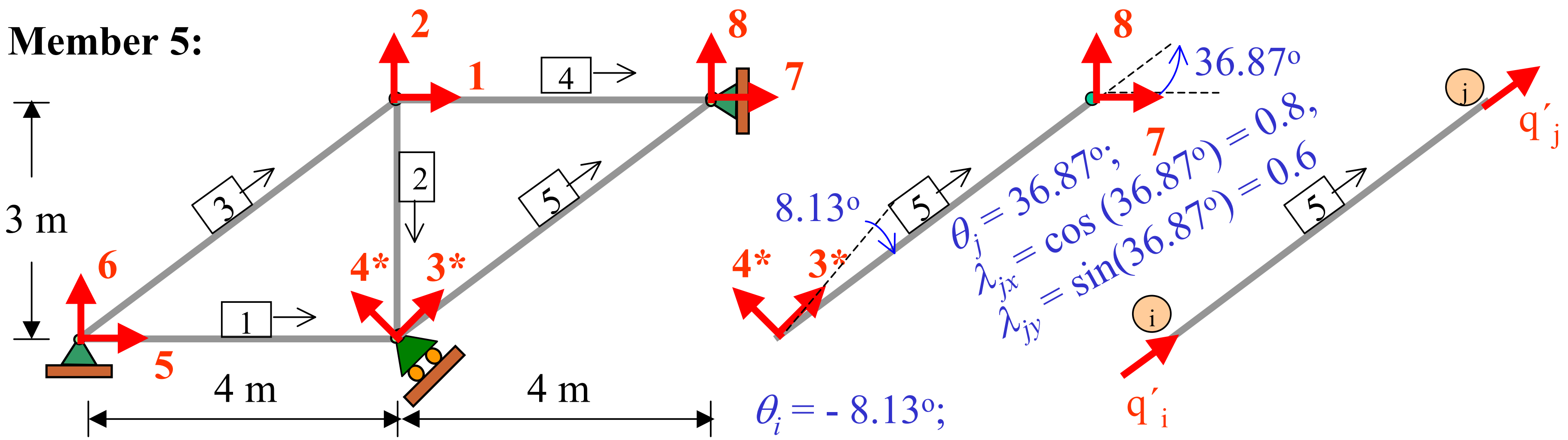


$$[k^*] = [T^T]\frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}[T]$$

$$[\mathrm{k}]_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \frac{AE}{4} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$[\mathrm{k}]_4 = AE \begin{array}{c} \\ 1 \\ 2 \\ 7 \\ 8 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 7 & 8 \end{array} \\ \begin{pmatrix} 0.25 & 0 & -0.25 & 0 \\ 0 & 0 & 0 & 0 \\ -0.25 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

Member 5:



$\theta_j = 36.87°;$
$\lambda_{jx} = \cos(36.87°) = 0.8,$
$\lambda_{jy} = \sin(36.87°) = 0.6$

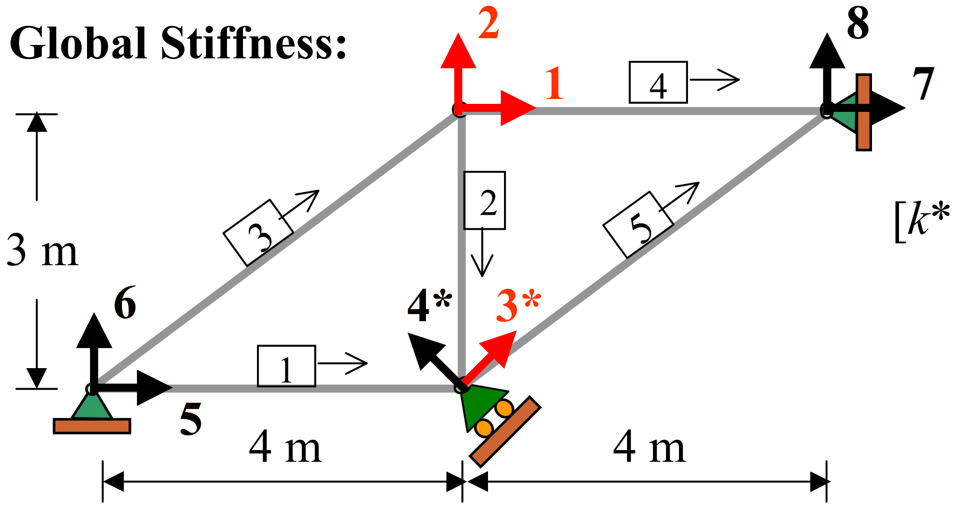
$\theta_i = -\ 8.13°;$
$\lambda_{ix} = \cos(-\ 8.13°) = 0.9899,$
$\lambda_{iy} = \sin(-\ 8.13°) = -0.1414$

$$[k^*] = [T^T]\frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}[T]$$

$$[k^*]_5 = \begin{pmatrix} 0.9899 & 0 \\ -0.1414 & 0 \\ 0 & 0.8 \\ 0 & 0.6 \end{pmatrix} \frac{AE}{5} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 0.9899 & -0.1414 & 0 & 0 \\ 0 & 0 & 0.8 & 0.6 \end{pmatrix}$$

$$[k^*]_5 = AE \begin{array}{c} \\ 3^* \\ 4^* \\ 7 \\ 8 \end{array} \begin{array}{c} \begin{array}{cccc} 3^* & 4^* & 7 & 8 \end{array} \\ \begin{pmatrix} 0.196 & -0.028 & -0.1584 & -0.1188 \\ -0.028 & 0.004 & 0.02263 & 0.01697 \\ -0.1584 & 0.02263 & 0.128 & 0.096 \\ -0.1188 & 0.01697 & 0.096 & 0.072 \end{pmatrix} \end{array}$$

Global Stiffness:



$$[k^*]_3 = AE \begin{array}{c} \\ 5 \\ 6 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{cccc} 5 & 6 & 1 & 2 \end{array} \\ \begin{pmatrix} 0.128 & 0.096 & -0.128 & -0.096 \\ 0.096 & 0.072 & -0.096 & -0.072 \\ -0.128 & -0.096 & 0.128 & 0.096 \\ -0.096 & -0.072 & 0.096 & 0.072 \end{pmatrix} \end{array}$$

$$[k]_4 = AE \begin{array}{c} \\ 1 \\ 2 \\ 7 \\ 8 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 7 & 8 \end{array} \\ \begin{pmatrix} 0.25 & 0 & -0.25 & 0 \\ 0 & 0 & 0 & 0 \\ -0.25 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

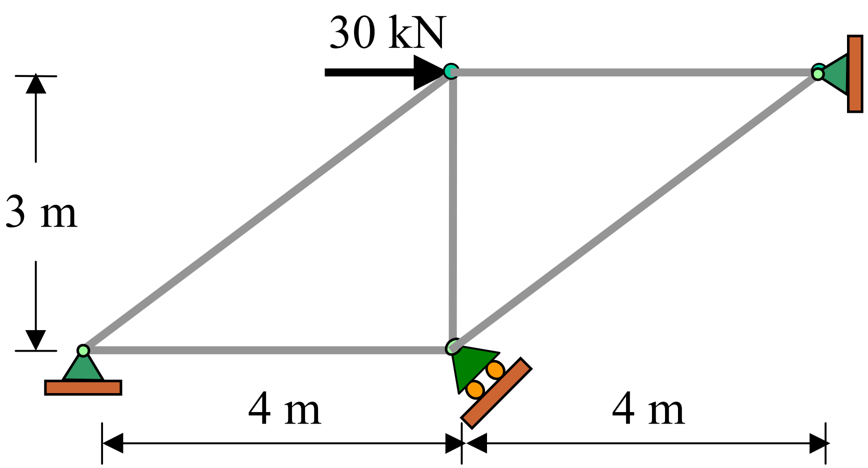
$$[k^*]_1 = AE \begin{array}{c} \\ 5 \\ 6 \\ 3^* \\ 4^* \end{array} \begin{array}{c} \begin{array}{cccc} 5 & 6 & 3^* & 4^* \end{array} \\ \begin{pmatrix} 0.25 & 0 & -0.1768 & 0.1768 \\ 0 & 0 & 0 & 0 \\ -0.1768 & 0 & 0.125 & -0.125 \\ 0.1768 & 0 & -0.125 & 0.125 \end{pmatrix} \end{array}$$

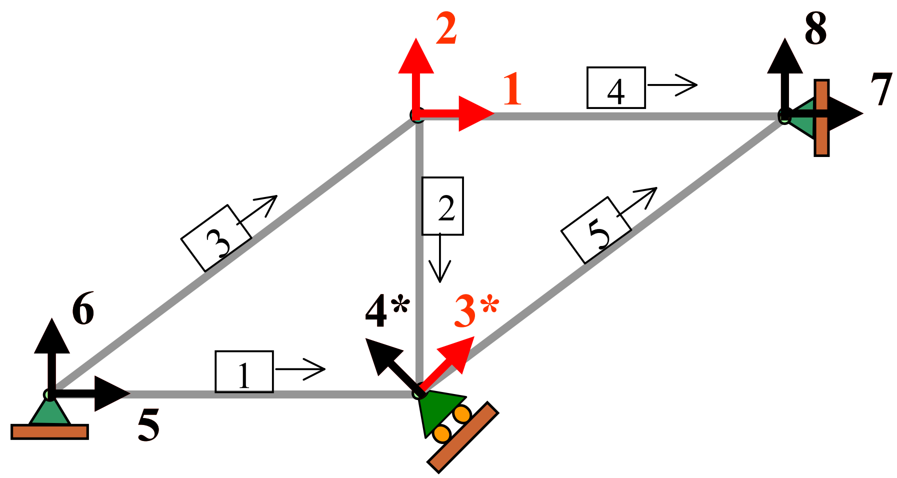
$$[k^*]_5 = AE \begin{array}{c} \\ 3^* \\ 4^* \\ 7 \\ 8 \end{array} \begin{array}{c} \begin{array}{cccc} 3^* & 4^* & 7 & 8 \end{array} \\ \begin{pmatrix} 0.196 & -0.028 & -0.1584 & -0.1188 \\ -0.028 & 0.004 & 0.02263 & 0.01697 \\ -0.1584 & 0.02263 & 0.128 & 0.096 \\ -0.1188 & 0.01697 & 0.096 & 0.072 \end{pmatrix} \end{array}$$

$$[k^*]_2 = AE \begin{array}{c} \\ 1 \\ 2 \\ 3^* \\ 4^* \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3^* & 4^* \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.3333 & -0.2357 & -0.2357 \\ 0 & -0.2357 & 0.1667 & 0.1667 \\ 0 & -0.2357 & 0.1667 & 0.1667 \end{pmatrix} \end{array}$$

$$[K] = AE \begin{array}{c} \\ 1 \\ 2 \\ 3^* \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3^* \end{array} \\ \begin{pmatrix} 0.378 & 0.096 & 0 \\ 0.096 & 0.4053 & -0.2357 \\ 0 & -0.2357 & 0.4877 \end{pmatrix} \end{array}$$

Global :



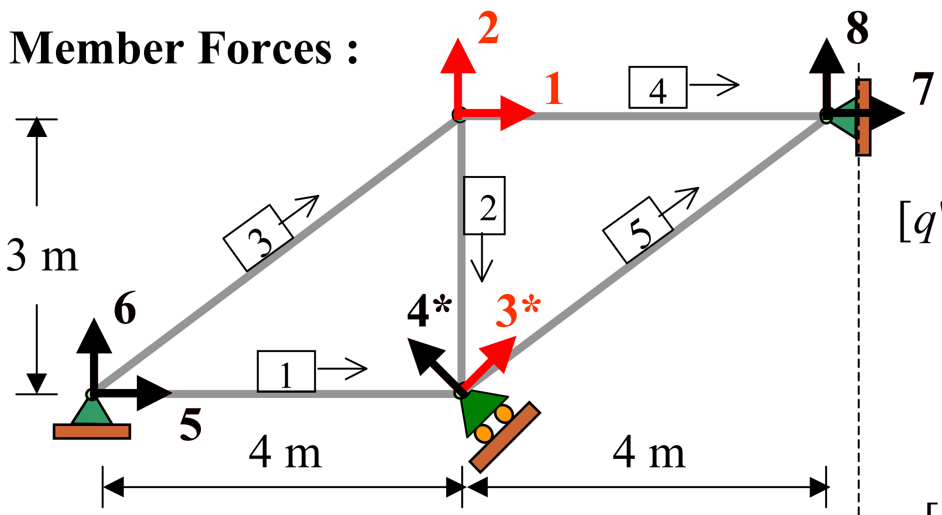


$$[Q] = [K][D] + [Q^F]$$

$$\begin{pmatrix} Q_1 = 30 \\ Q_2 = 0 \\ Q_{3*} = 0 \end{pmatrix} = AE \begin{array}{c} \\ 1 \\ 2 \\ 3^* \end{array}\!\!\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3^* \end{array} \\ \begin{pmatrix} 0.378 & 0.096 & 0 \\ 0.096 & 0.4053 & -0.2357 \\ 0 & -0.2357 & 0.4877 \end{pmatrix} \end{array} \begin{pmatrix} D_1 \\ D_2 \\ D_{3*} \end{pmatrix}$$

$$\begin{pmatrix} D_1 \\ D_2 \\ D_{3*} \end{pmatrix} = \frac{1}{AE} \begin{pmatrix} 86.612 \\ -28.535 \\ -13.791 \end{pmatrix}$$

Member Forces :



$$\begin{pmatrix} D_1 \\ D_2 \\ D_{3*} \end{pmatrix} = \frac{1}{AE}\begin{pmatrix} 86.612 \\ -28.535 \\ -13.791 \end{pmatrix}$$

Member	λ_{ix}	λ_{iy}	λ_{jx}	λ_{jy}
#1	1	0	0.707	-0.707
#2	0	-1	-0.707	-0.707
#3	0.8	0.6	0.8	0.6

$$[q'_F]_m = \frac{AE}{L}\begin{bmatrix} -\lambda_x & -\lambda_y & \lambda_x & \lambda_x \end{bmatrix}\begin{bmatrix} D_{xi} \\ D_{yi} \\ D_{xj} \\ D_{yj} \end{bmatrix} + \left[q'^F\right]$$

$$[q'_F]_1 = \frac{AE}{4}\begin{bmatrix} -1 & 0 & 0.707 & -0.707 \end{bmatrix}\begin{pmatrix} 0 \\ 0 \\ D_{3*} \\ 0 \end{pmatrix}$$

$= -2.44$ kN, (C)

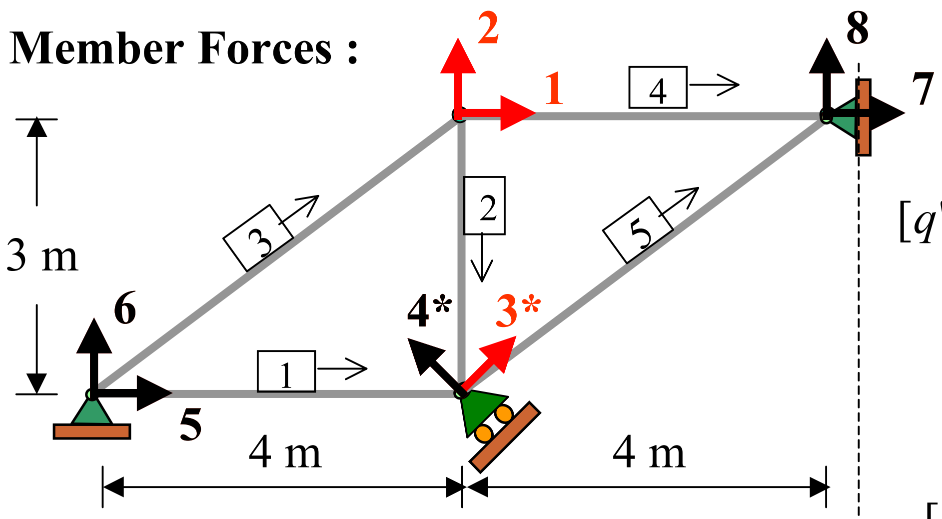
$$[q'_F]_2 = \frac{AE}{3}\begin{bmatrix} 0 & 1 & -0.707 & -0.707 \end{bmatrix}\begin{pmatrix} D_1 \\ D_2 \\ D_{3*} \\ 0 \end{pmatrix}$$

$= -6.26$ kN, (C)

$$[q'_F]_3 = \frac{AE}{5}\begin{bmatrix} -0.8 & -0.6 & 0.8 & 0.6 \end{bmatrix}\begin{pmatrix} 0 \\ 0 \\ D_1 \\ D_2 \end{pmatrix}$$

$= 10.43$ kN, (T)

Member Forces :



$$\begin{pmatrix} D_1 \\ D_2 \\ D_{3*} \end{pmatrix} = \frac{1}{AE}\begin{pmatrix} 86.612 \\ -28.535 \\ -13.791 \end{pmatrix}$$

Member	λ_{ix}	λ_{iy}	λ_{jx}	λ_{jy}
#4	1	0	1	0
#5	0.9899	-0.141	0.8	0.6

$$[q'_F]_m = \frac{AE}{L}\begin{bmatrix} -\lambda_x & -\lambda_y & \lambda_x & \lambda_x \end{bmatrix}\begin{bmatrix} D_{xi} \\ D_{yi} \\ D_{xj} \\ D_{yj} \end{bmatrix} + \left[q'^F\right]$$

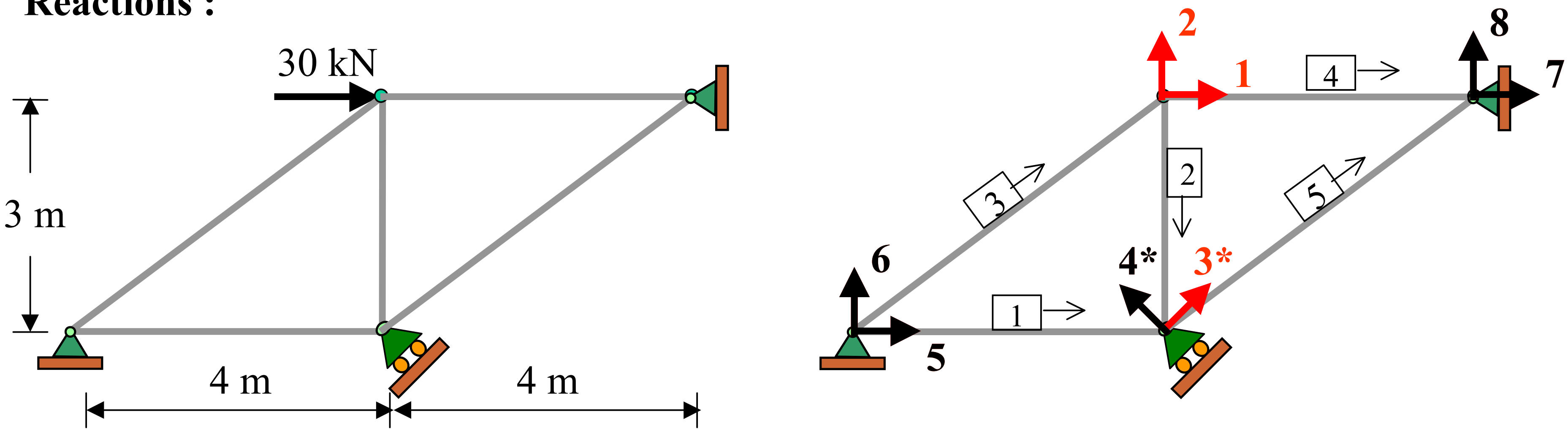
$$[q'_F]_4 = \frac{AE}{4}\begin{bmatrix} -1 & 0 & 1 & 0 \end{bmatrix}\begin{pmatrix} D_1 \\ D_2 \\ 0 \\ 0 \end{pmatrix}$$

$$= -21.65 \text{ kN, (C)}$$

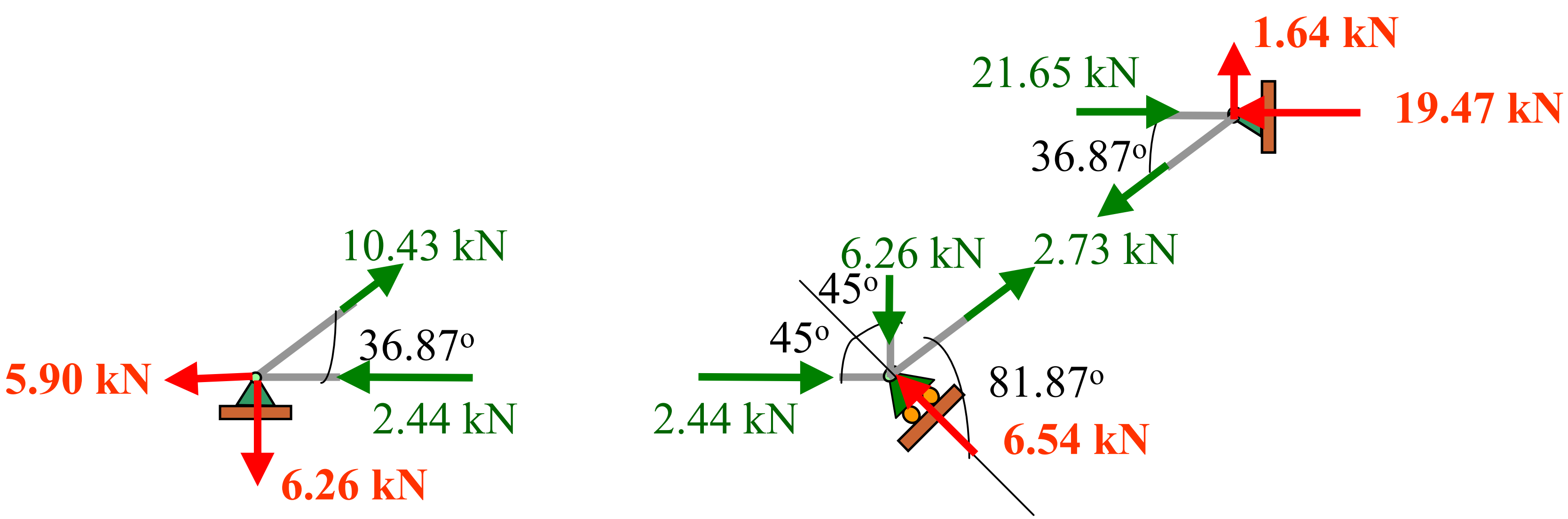
$$[q'_F]_5 = \frac{AE}{5}\begin{bmatrix} -0.9899 & 0.141 & 0.8 & 0.6 \end{bmatrix}\begin{pmatrix} D_{3*} \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$= 2.73 \text{ kN, (T)}$$

Reactions :

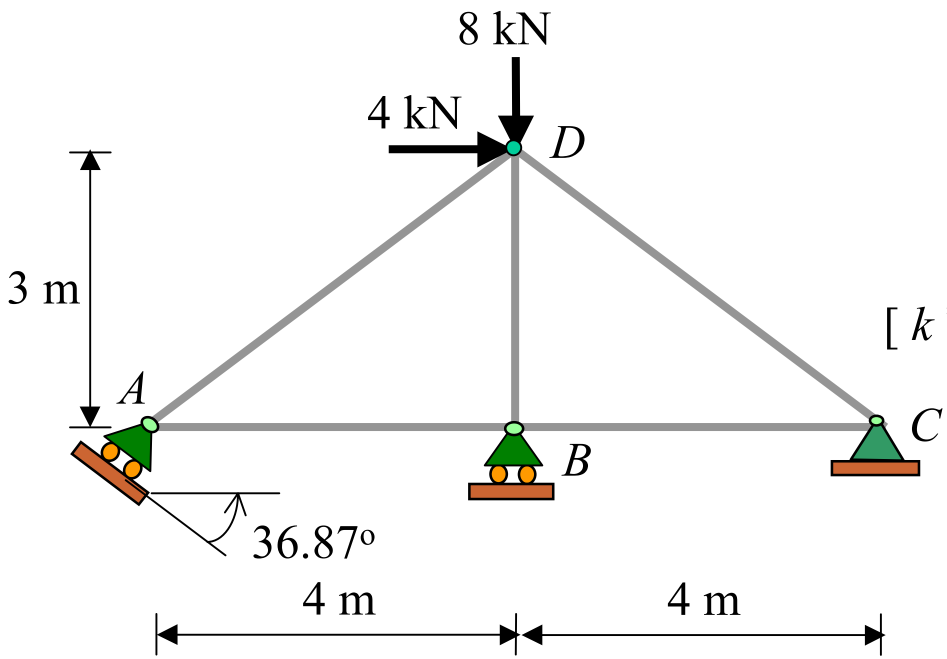


Member	$[q']_1$	$[q']_2$	$[q']_3$	$[q']_4$	$[q']_5$
Member Force (kN)	-2.44	-6.26	10.43	-21.65	2.73

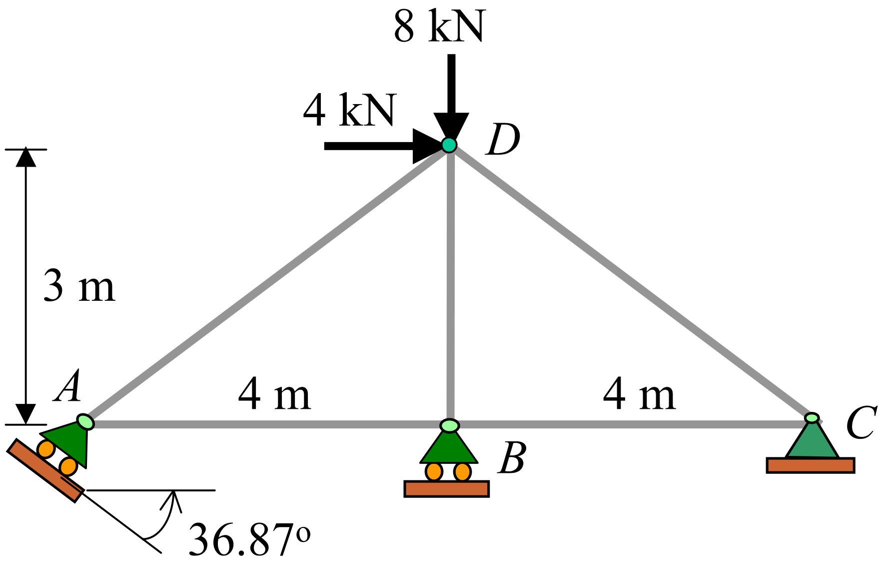


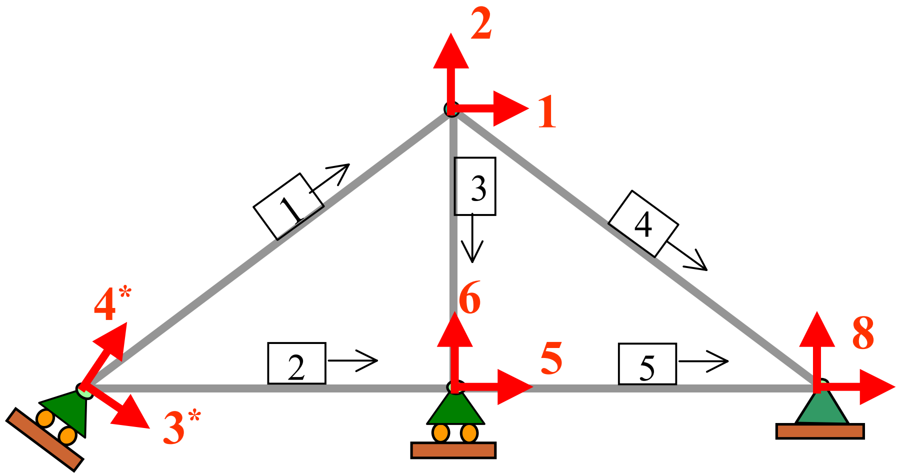
Example 7

For the truss shown, use the stiffness method to:
(b) Determine the **end forces** of each member and **reactions** at supports.
(a) Determine the **displacement** of the loaded joint.
Take AE = 8(10^3) kN.



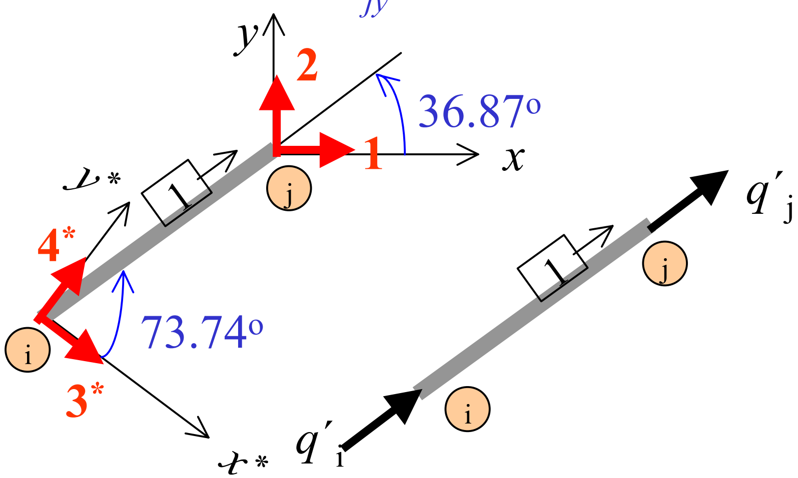
$$[k^*]_m = \frac{AE}{L}\begin{array}{c} \\ U_i \\ V_i \\ U_j \\ V_j \end{array}\begin{array}{c} \begin{array}{cccc} U_i & V_i & U_j & V_j \end{array} \\ \begin{pmatrix} \lambda_{ix}\lambda_{ix} & \lambda_{ix}\lambda_{iy} & -\lambda_{ix}\lambda_{jx} & -\lambda_{ix}\lambda_{jy} \\ \lambda_{iy}\lambda_{ix} & \lambda_{iy}\lambda_{iy} & -\lambda_{iy}\lambda_{jx} & -\lambda_{iy}\lambda_{jy} \\ -\lambda_{jx}\lambda_{ix} & -\lambda_{jx}\lambda_{iy} & \lambda_{jx}\lambda_{jx} & \lambda_{jx}\lambda_{jy} \\ -\lambda_{jy}\lambda_{ix} & -\lambda_{jy}\lambda_{iy} & \lambda_{jy}\lambda_{jx} & \lambda_{jy}\lambda_{jy} \end{pmatrix} \end{array}$$





Member 1:

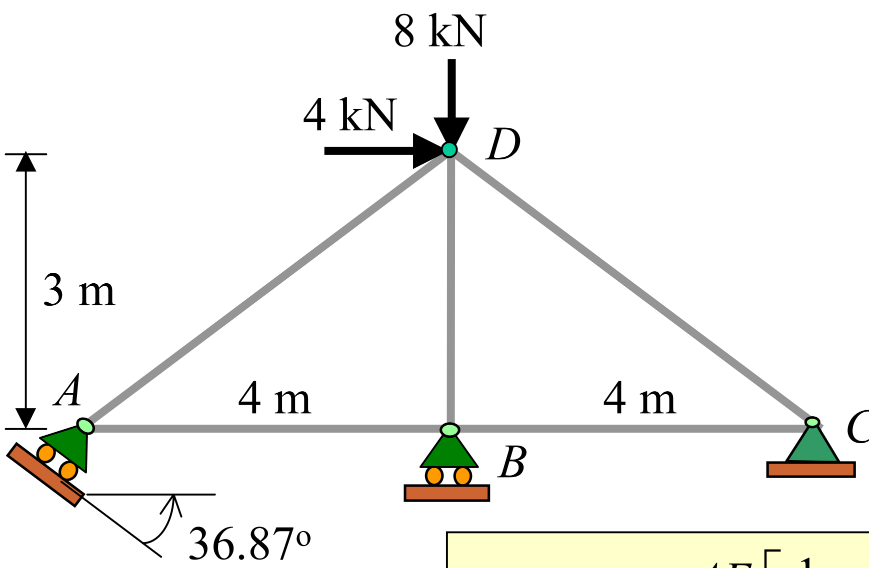
$\lambda_{jx} = \cos 36.87^{\circ} = 0.8,$
$\lambda_{jy} = \sin 36.87^{\circ} = 0.6$

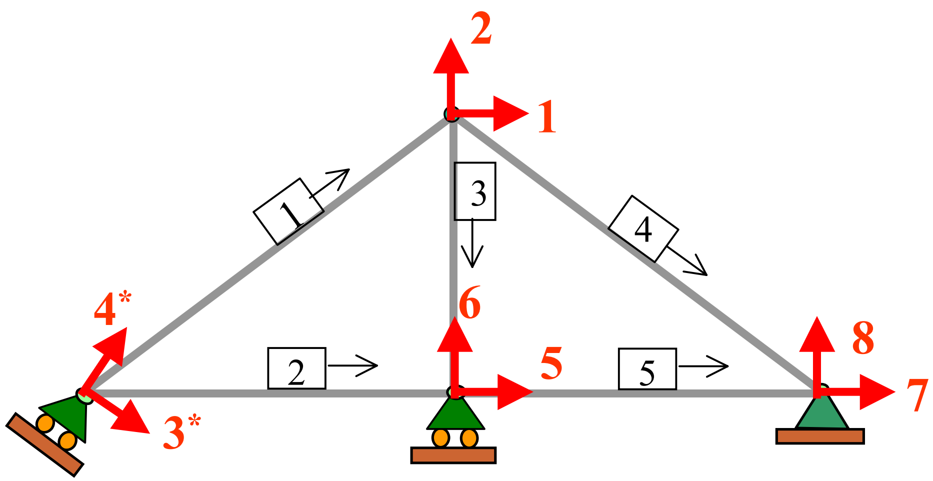


$$\boxed{[q^{*}] = [T^{*}]^{\mathrm{T}}[q'] + [T^{*}]^{\mathrm{T}}[q'^{\mathrm{F}}]}$$

$$\begin{pmatrix} q_{3*} \\ q_{4*} \\ q_1 \\ q_2 \end{pmatrix} = \underbrace{\begin{pmatrix} 0.28 & 0 \\ 0.96 & 0 \\ 0 & 0.8 \\ 0 & 0.6 \end{pmatrix}}_{[T^{*}]^{\mathrm{T}}} \begin{pmatrix} q'_{\mathrm{i}} \\ q'_{\mathrm{j}} \end{pmatrix}$$

$\lambda_{ix} = \cos 73.74^{\circ} = 0.28,$
$\lambda_{iy} = \sin 73.74^{\circ} = 0.96$

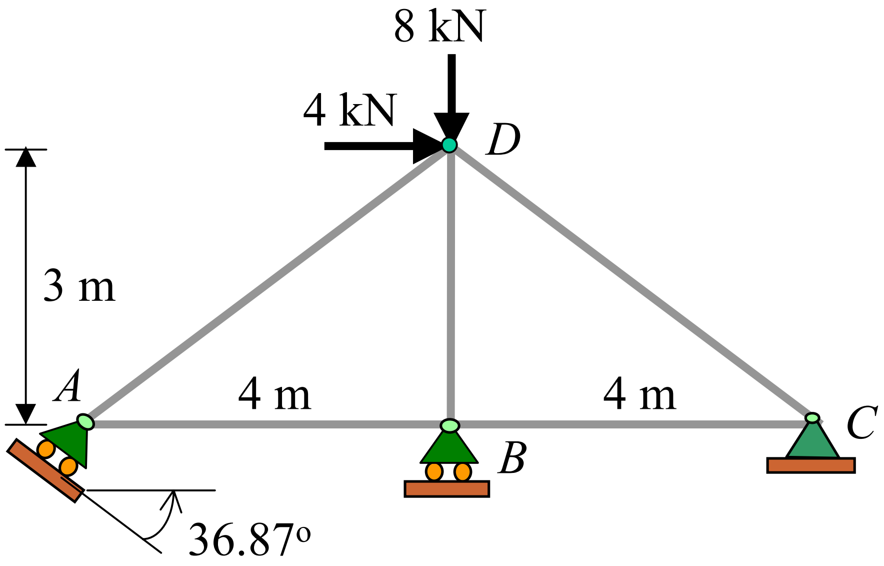


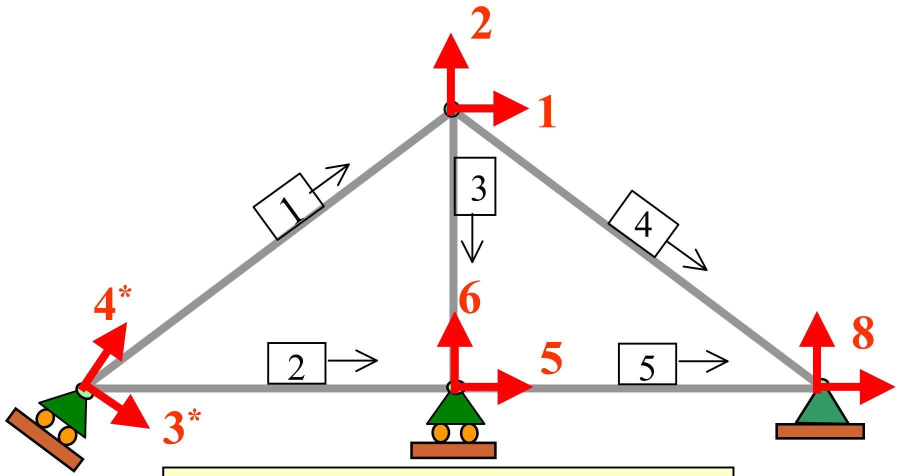


$$[k] = [T^T]\frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}[T]$$

$$[k]_1 = \begin{pmatrix} 0.28 & 0 \\ 0.96 & 0 \\ 0 & 0.8 \\ 0 & 0.6 \end{pmatrix} \frac{8\text{x}10^3}{5} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 0.28 & 0.96 & 0 & 0 \\ 0 & 0 & 0.8 & 0.6 \end{bmatrix}$$

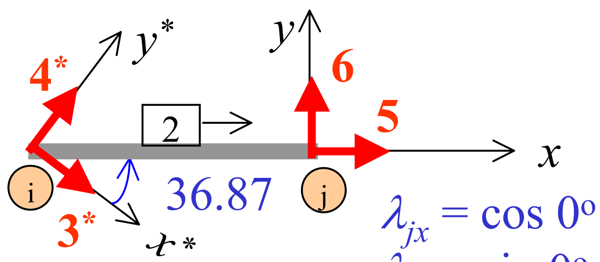
$$[k]_1 = 8\text{x}10^3 \begin{array}{c} \\ 3^* \\ 4^* \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{cccc} 3^* & 4^* & 1 & 2 \end{array} \\ \begin{pmatrix} 0.01568 & 0.05376 & -0.0448 & -0.0336 \\ 0.05376 & 0.18432 & -0.1536 & -0.1152 \\ -0.0448 & -0.1536 & 0.128 & 0.096 \\ -0.0336 & -0.1152 & 0.096 & 0.072 \end{pmatrix} \end{array}$$





$$[q^*] = [T^*]^T[q'] + [T^*]^T[q'^F]$$

Member 2:

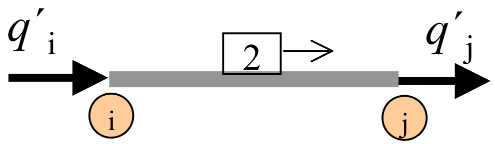


$\lambda_{jx} = \cos 0^o = 1,$
$\lambda_{jy} = \sin 0^o = 0$

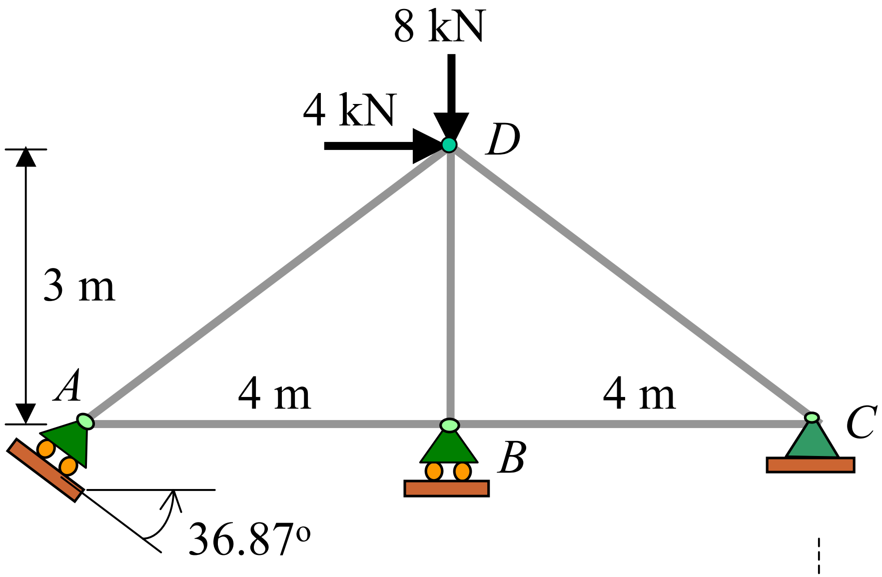
$\lambda_{ix} = \cos 36.87^o = 0.8,$
$\lambda_{iy} = \sin 36.87^o = 0.6$

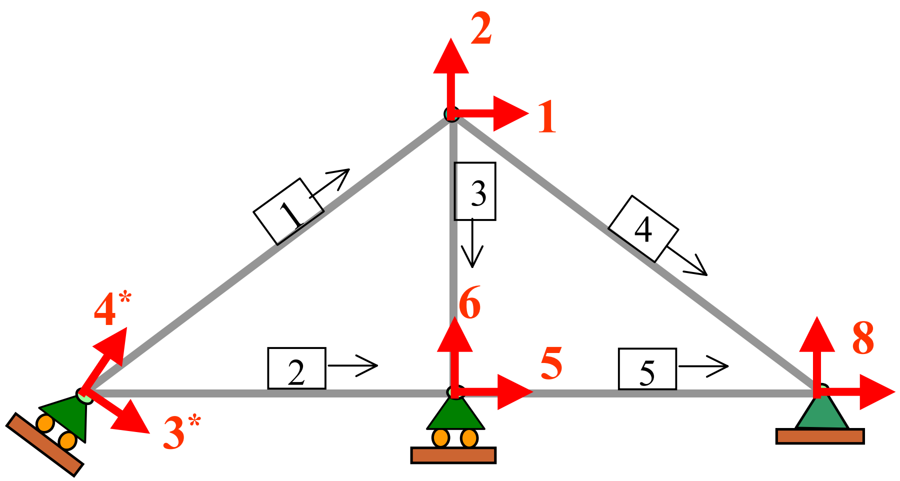
$$\begin{pmatrix} q_{3*} \\ q_{4*} \\ q_5 \\ q_6 \end{pmatrix} = \underbrace{\begin{pmatrix} 0.8 & 0 \\ 0.6 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}}_{[T^*]^T} \begin{pmatrix} q'_i \\ q'_j \end{pmatrix}$$

$$[k] = [T^T]\ \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}[T]$$



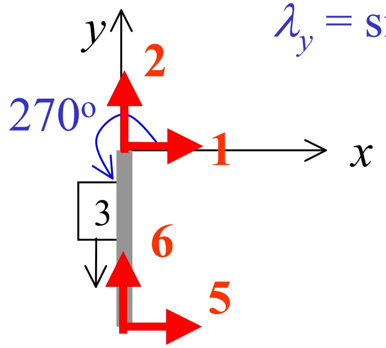
$$[k]_2 = 8\text{x}10^3 \begin{array}{c} \\ 3^* \\ 4^* \\ 5 \\ 6 \end{array}\begin{array}{c} \begin{array}{cccc} 3^* & 4^* & 5 & 6 \end{array} \\ \begin{pmatrix} 0.16 & 0.12 & -0.2 & 0 \\ 0.12 & 0.09 & -0.15 & 0 \\ -0.2 & -0.15 & 0.25 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$





Member 3:

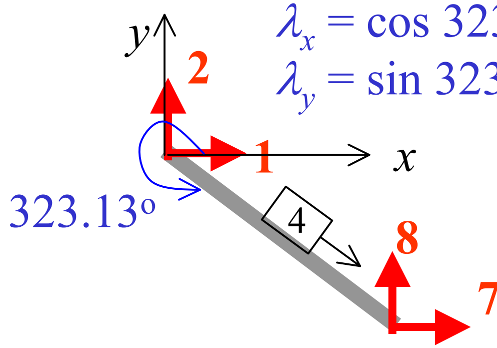
$\lambda_x = \cos 270^\circ = 0,$
$\lambda_y = \sin 270^\circ = -1$



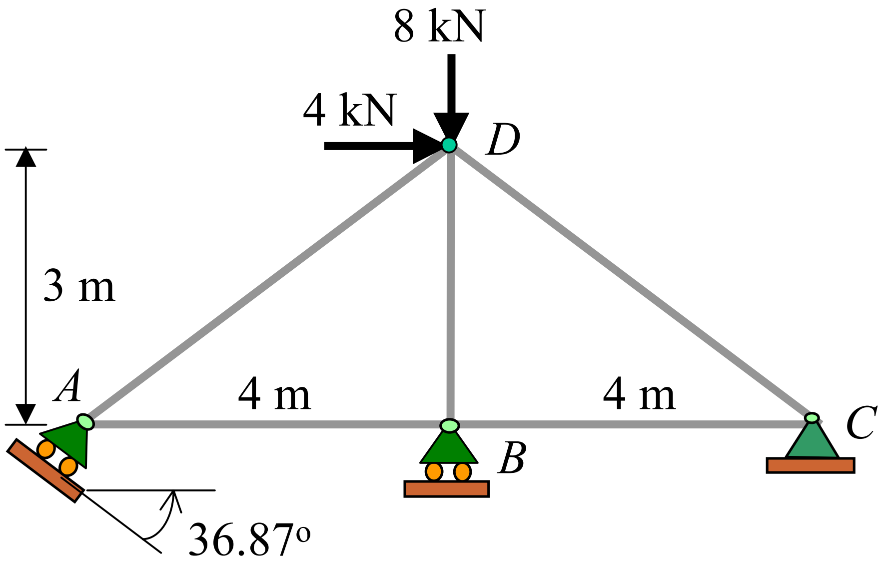
$$[k]_3 = 8\text{x}10^3 \begin{array}{c} \\ 1 \\ 2 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 5 & 6 \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.333 & 0 & -0.333 \\ 0 & 0 & 0 & 0 \\ 0 & -0.333 & 0 & 0.333 \end{pmatrix} \end{array}$$

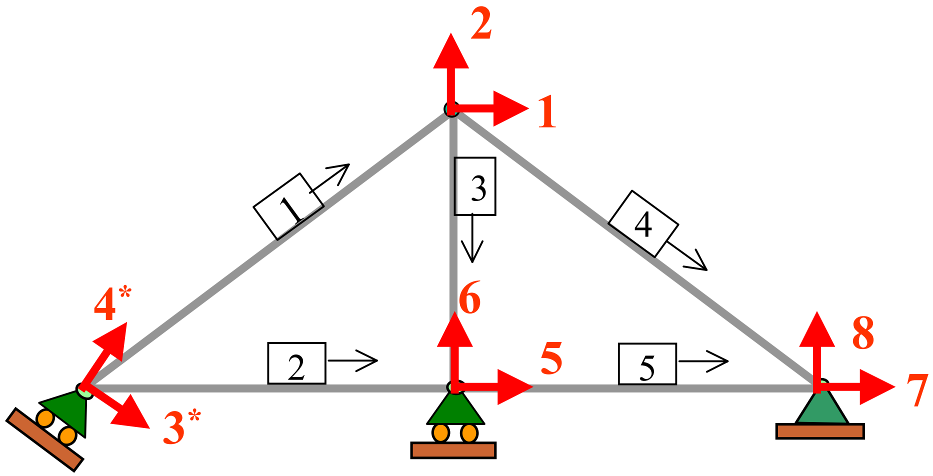
Member 4:

$\lambda_x = \cos 323.13^\circ = 0.8,$
$\lambda_y = \sin 323.13^\circ = -0.6$

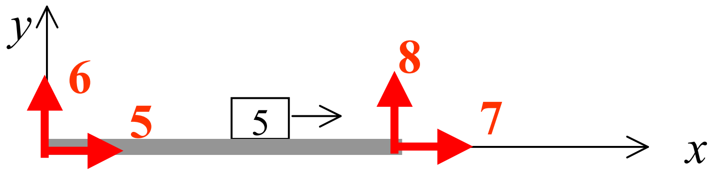


$$[k]_4 = 8\text{x}10^3 \begin{array}{c} \\ 1 \\ 2 \\ 7 \\ 8 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 7 & 8 \end{array} \\ \begin{pmatrix} 0.128 & -0.096 & -0.128 & 0.096 \\ -0.096 & 0.072 & 0.096 & -0.072 \\ -0.128 & 0.096 & 0.128 & -0.096 \\ 0.096 & -0.072 & -0.096 & 0.072 \end{pmatrix} \end{array}$$





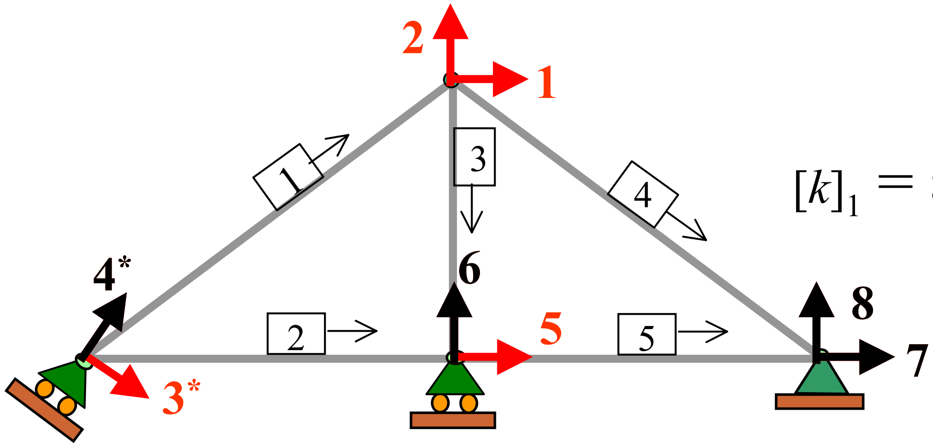
Member 5:



$\lambda_x = \cos 0^{\circ} = 1,$

$\lambda_y = \sin 0^{\circ} = 0$

$$[k]_5 = 8\text{x}10^3 \begin{array}{c} \\ 5 \\ 6 \\ 7 \\ 8 \end{array} \begin{array}{c} \begin{array}{cccc} 5 & 6 & 7 & 8 \end{array} \\ \begin{pmatrix} 0.25 & 0 & -0.25 & 0 \\ 0 & 0 & 0 & 0 \\ -0.25 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$



$$[k]_1 = 8\text{x}10^3 \quad \begin{array}{cc} & \begin{array}{cccc} 3^* & 4^* & 1 & 2 \end{array} \\ \begin{array}{c} 3^* \\ 4^* \\ 1 \\ 2 \end{array} & \begin{pmatrix} 0.01568 & 0.05376 & -0.0448 & -0.0336 \\ 0.05376 & 0.18432 & -0.1536 & -0.1152 \\ -0.0448 & -0.1536 & 0.128 & 0.096 \\ -0.0336 & -0.1152 & 0.096 & 0.072 \end{pmatrix} \end{array}$$

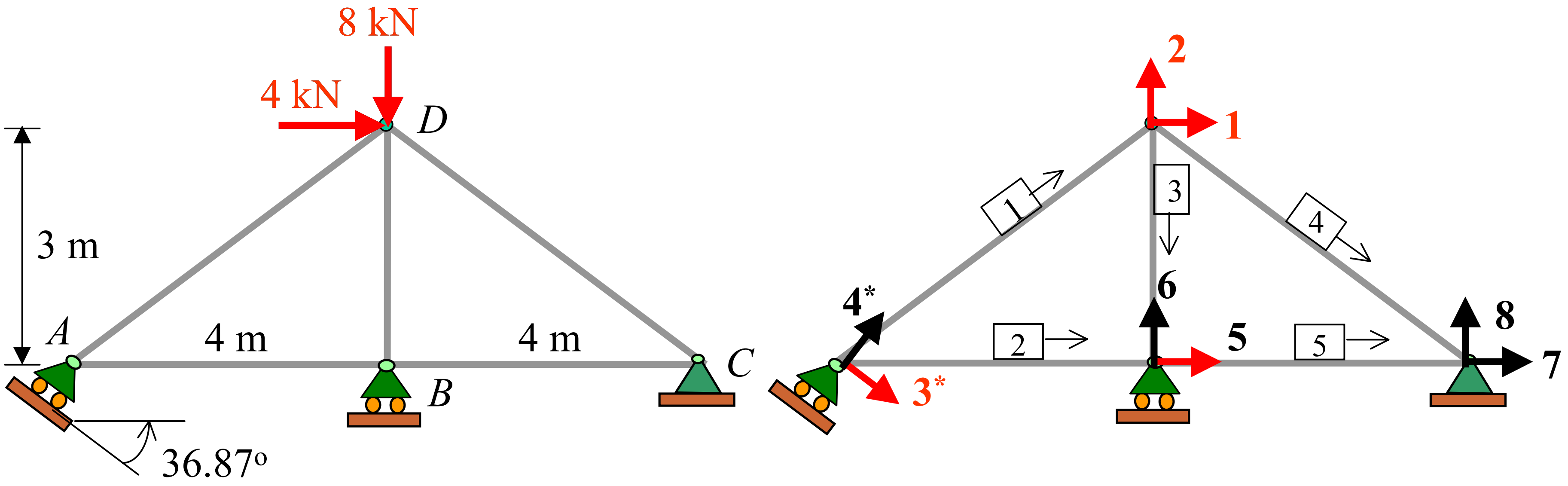
$$[k]_2 = 8\text{x}10^3 \quad \begin{array}{cc} & \begin{array}{cccc} 3^* & 4^* & 5 & 6 \end{array} \\ \begin{array}{c} 3^* \\ 4^* \\ 5 \\ 6 \end{array} & \begin{pmatrix} 0.16 & 0.12 & -0.2 & 0 \\ 0.12 & 0.09 & -0.15 & 0 \\ -0.2 & -0.15 & 0.25 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

$$[k]_3 = 8\text{x}10^3 \quad \begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 5 & 6 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 5 \\ 6 \end{array} & \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.333 & 0 & -0.333 \\ 0 & 0 & 0 & 0 \\ 0 & -0.333 & 0 & 0.333 \end{pmatrix} \end{array}$$

$$[k]_4 = 8\text{x}10^3 \quad \begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 7 & 8 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 7 \\ 8 \end{array} & \begin{pmatrix} 0.128 & -0.096 & -0.128 & 0.096 \\ -0.096 & 0.072 & 0.096 & -0.072 \\ -0.128 & 0.096 & 0.128 & -0.096 \\ 0.096 & -0.072 & -0.096 & 0.072 \end{pmatrix} \end{array}$$

$$[k]_5 = 8\text{x}10^3 \quad \begin{array}{cc} & \begin{array}{cccc} 5 & 6 & 7 & 8 \end{array} \\ \begin{array}{c} 5 \\ 6 \\ 7 \\ 8 \end{array} & \begin{pmatrix} 0.25 & 0 & -0.25 & 0 \\ 0 & 0 & 0 & 0 \\ -0.25 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

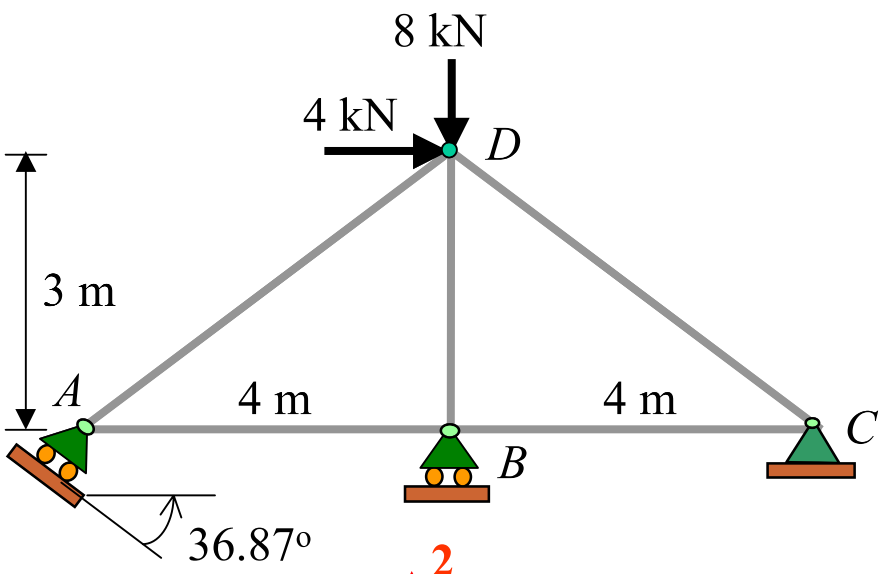
$$[K] = 8\text{x}10^3 \quad \begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3^* & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3^* \\ 5 \end{array} & \begin{pmatrix} 0.256 & 0.0 & -0.0448 & 0 \\ 0.0 & 0.474 & -0.0336 & 0 \\ -0.0448 & -0.0336 & 0.17568 & -0.2 \\ 0 & 0 & -0.2 & 0.5 \end{pmatrix} \end{array}$$

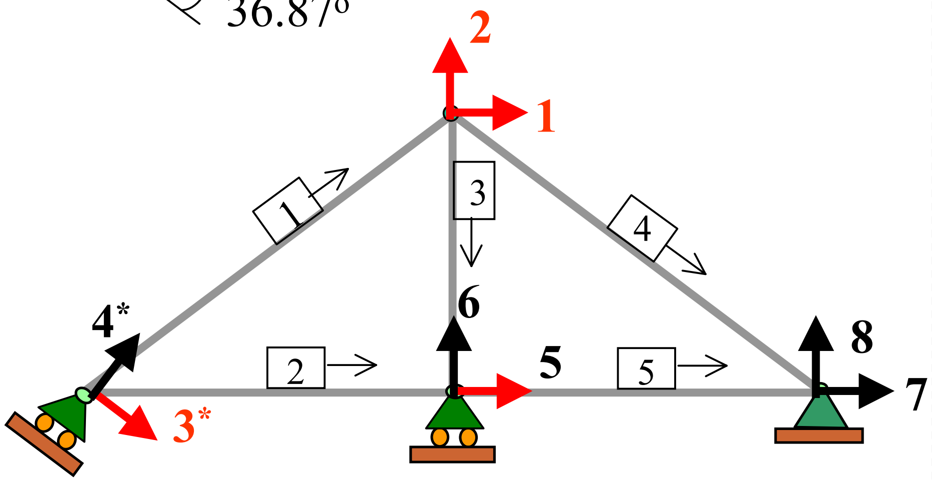


Global: $\boxed{[Q] = [K][D] + [Q^F]}$

$$\begin{bmatrix} Q_1 = 4 \\ Q_2 = -8 \\ Q_{3*} = 0 \\ Q_5 = 0 \end{bmatrix} = 8\text{x}10^3 \begin{array}{c} \\ 1 \\ 2 \\ 3^* \\ 5 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3^* & 5 \end{array} \\ \begin{bmatrix} 0.256 & 0.0 & -0.0448 & 0 \\ 0.0 & 0.474 & -0.0336 & 0 \\ -0.0448 & -0.0336 & 0.17568 & -0.2 \\ 0 & 0 & -0.2 & 0.5 \end{bmatrix} \end{array} \begin{bmatrix} D_1 \\ D_2 \\ D_{3*} \\ D_5 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \\ D_{3*} \\ D_5 \end{bmatrix} = \begin{bmatrix} 1.988\text{x}10^{-3} & \text{m} \\ -2.0824\text{x}10^{-3} & \text{m} \\ 1.996\text{x}10^{-4} & \text{m} \\ 7.984\text{x}10^{-5} & \text{m} \end{bmatrix}$$





$$\begin{pmatrix} D_1 \\ D_2 \\ D_{3*} \\ D_5 \end{pmatrix} = \begin{pmatrix} 1.988\text{x}10^{-3} & \text{m} \\ -2.0824\text{x}10^{-3} & \text{m} \\ 1.996\text{x}10^{-4} & \text{m} \\ 7.984\text{x}10^{-5} & \text{m} \end{pmatrix}$$

Member forces

$$[q'_F]_m = \frac{AE}{L}\begin{bmatrix} -\lambda_x & -\lambda_y & \lambda_x & \lambda_x \end{bmatrix}\begin{bmatrix} D_{xi} \\ D_{yi} \\ D_{xj} \\ D_{yj} \end{bmatrix} + \left[q'^F\right]$$

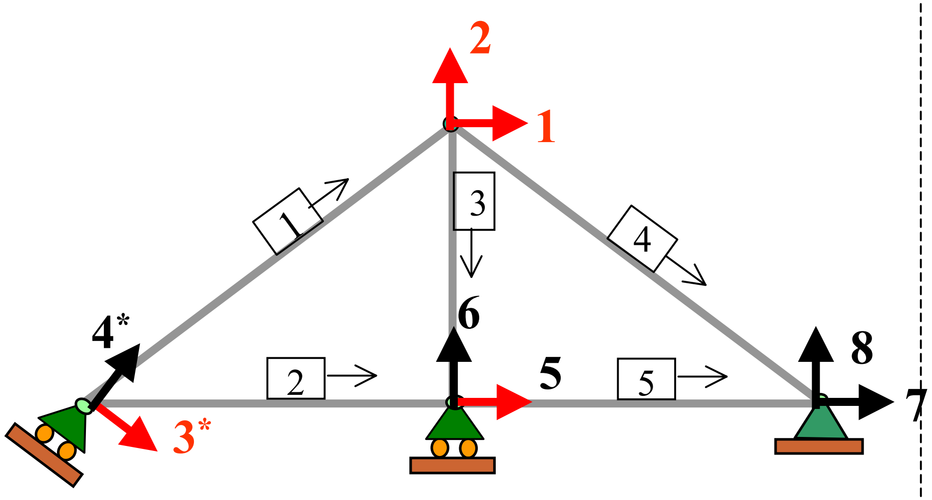
Member	λ_{ix}	λ_{iy}	λ_{jx}	λ_{jy}
#1	0.28	0.96	0.8	0.6
#2	0.8	0.6	1	0

$$[q'_F]_1 = \frac{8\text{x}10^3}{5}\begin{bmatrix} -0.28 & -0.96 & 0.8 & 0.6 \end{bmatrix}\begin{pmatrix} D_{3*} \\ 0 \\ D_1 \\ D_2 \end{pmatrix}$$

$= 0.46$ kN, (T)

$$[q'_F]_2 = \frac{8\text{x}10^3}{4}\begin{bmatrix} -0.8 & -0.6 & 1 & 0 \end{bmatrix}\begin{pmatrix} D_{3*} \\ 0 \\ D_5 \\ 0 \end{pmatrix}$$

$= -0.16$ kN, (C)



$$
\begin{pmatrix} D_1 \\ D_2 \\ D^*_3 \\ D_5 \end{pmatrix} = \begin{pmatrix} 1.988\text{x}10^{-3} & \text{m} \\ -2.0824\text{x}10^{-3} & \text{m} \\ 1.996\text{x}10^{-4} & \text{m} \\ 7.984\text{x}10^{-5} & \text{m} \end{pmatrix}
$$

Member	λ_{ix}	λ_{iy}	λ_{jx}	λ_{jy}
#3	0	-1	0	-1
#4	0.8	-0.6	0.8	-0.6
#5	1	0	1	0

$$
[q'_F]_m = \frac{AE}{L}\begin{bmatrix} -\lambda_x & -\lambda_y & \lambda_x & \lambda_x \end{bmatrix}\begin{bmatrix} D_{xi} \\ D_{yi} \\ D_{xj} \\ D_{yj} \end{bmatrix} + \begin{bmatrix} q'^F \end{bmatrix}
$$

$$
[q'_F]_3 = \frac{8\text{x}10^3}{3}\begin{bmatrix} 0 & 1 & 0 & -1 \end{bmatrix}\begin{pmatrix} D_1 \\ D_2 \\ D_5 \\ 0 \end{pmatrix}
$$

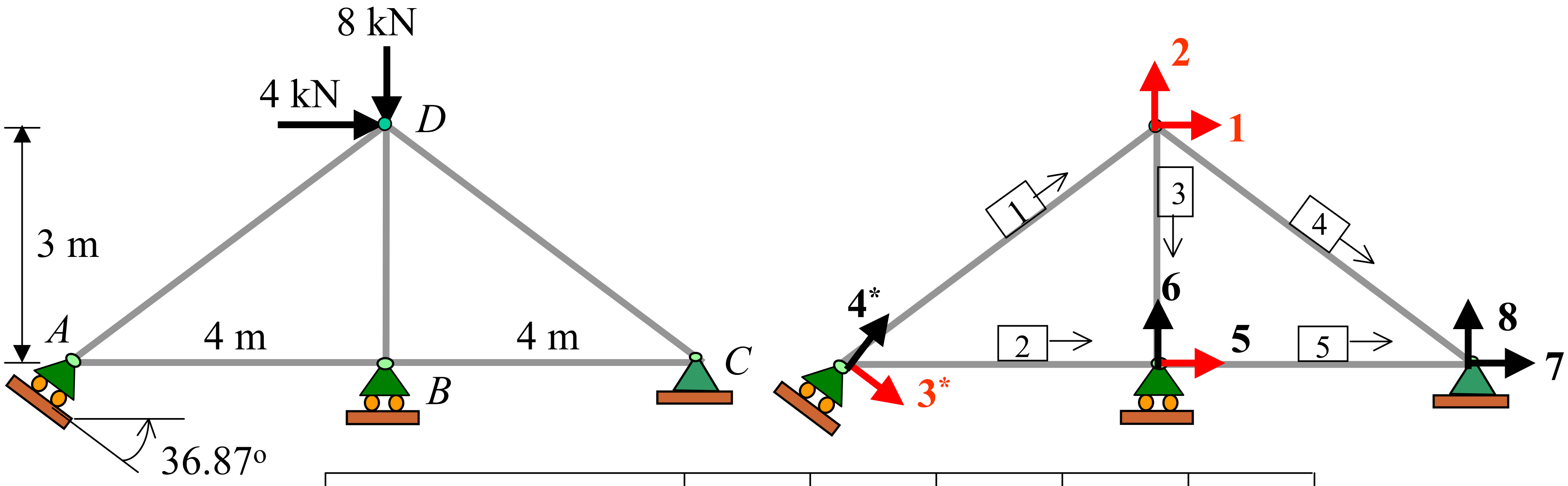
$$
= -5.55 \text{ kN}
$$

$$
[q'_F]_4 = \frac{8\text{x}10^3}{5}\begin{bmatrix} -0.8 & 0.6 & 0.8 & -0.6 \end{bmatrix}\begin{pmatrix} D_1 \\ D_2 \\ 0 \\ 0 \end{pmatrix}
$$

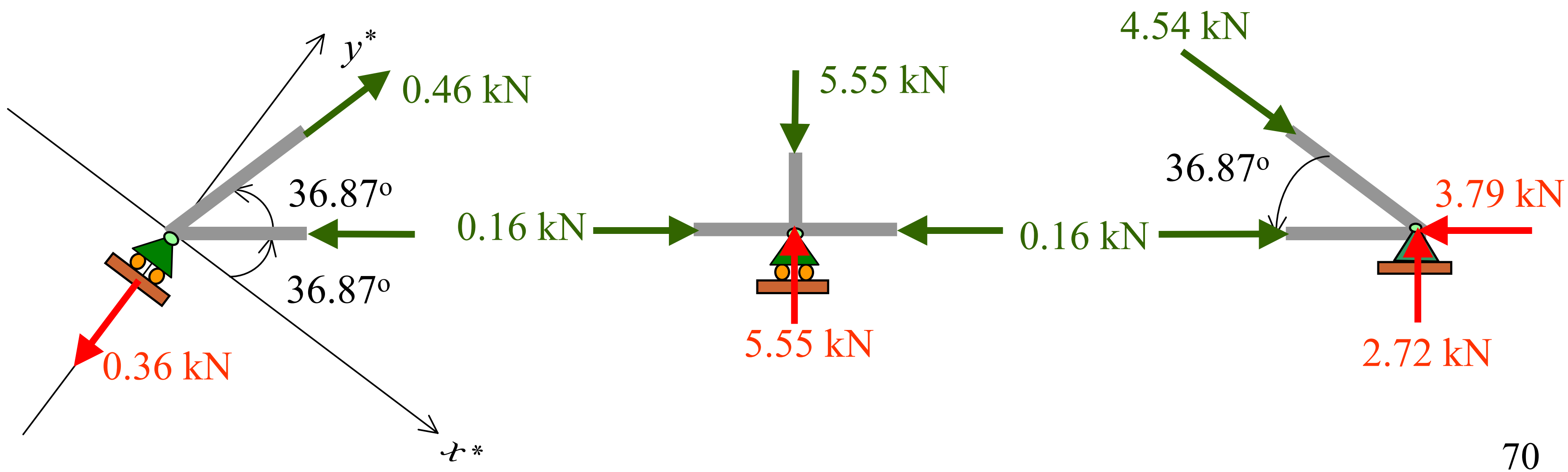
$$
= -4.54 \text{ kN}
$$

$$
[q'_F]_5 = \frac{8\text{x}10^3}{4}\begin{bmatrix} -1 & 0 & 1 & 0 \end{bmatrix}\begin{pmatrix} D_5 \\ 0 \\ 0 \\ 0 \end{pmatrix}
$$

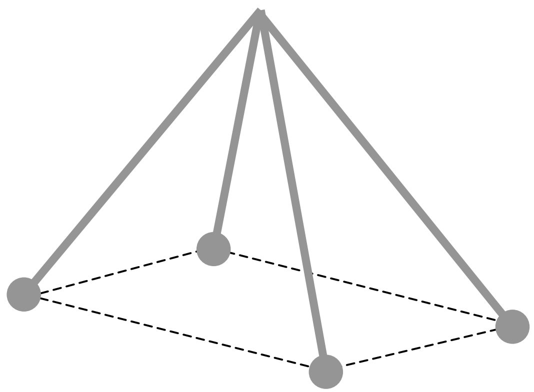
$$
= -0.16 \text{ kN}
$$



Member	$[q]_1$	$[q]_2$	$[q]_3$	$[q]_4$	$[q]_5$
Member Force (kN)	0.46	-0.16	-5.55	-4.54	-0.16



Space-Truss Analysis



Member Local Stiffness [*k´*]:

$$[q'] = [k'][d'] + [q'^{F}]$$

$$= [k'][T][d] + [q'^{F}]$$

$$\begin{bmatrix} q'_i \\ q'_j \end{bmatrix} = \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} d'_i \\ d'_j \end{bmatrix} + \begin{bmatrix} q'^{F}_i \\ q'^{F}_j \end{bmatrix}$$

$$= \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}[T]\begin{bmatrix} d_{ix} \\ d_{iy} \\ d_{iz} \\ d_{jx} \\ d_{jy} \\ d_{jz} \end{bmatrix} + [q'^{F}]$$

where,

$$[T] = \begin{bmatrix} \lambda_x & \lambda_y & \lambda_z & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_x & \lambda_y & \lambda_z \end{bmatrix}$$

Member Global Stiffness [k_m]:

$$[k_m] = [T]^T [k'] \, [T]$$

$$\left[k_m\right] = \begin{bmatrix} \lambda_x & 0 \\ \lambda_y & 0 \\ \lambda_z & 0 \\ 0 & \lambda_x \\ 0 & \lambda_y \\ 0 & \lambda_z \end{bmatrix} \frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \lambda_x & \lambda_y & \lambda_z & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_x & \lambda_y & \lambda_z \end{bmatrix}$$

Global equilibrium matrix:

$$\boxed{[Q] = [K][D] + [Q^{\mathrm{F}}]}$$

Joint Load

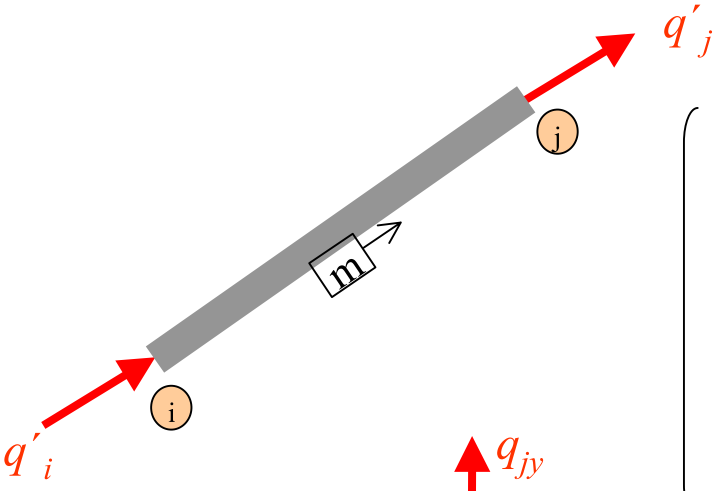
Unknown Displacement

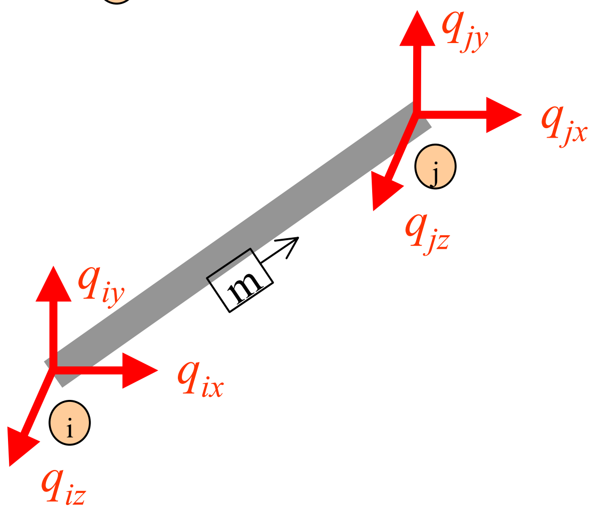
Fixed End Forces

$$\left[\begin{array}{c} Q_I \\ \hdashline Q_{II} \end{array}\right] = \left[\begin{array}{c:c} \mathbf{K_{I,I}} & \mathbf{K_{I,II}} \\ \hdashline \mathbf{K_{II,I}} & \mathbf{K_{II,II}} \end{array}\right] \left[\begin{array}{c} D_u \\ \hdashline D_k \end{array}\right] + \left[\begin{array}{c} Q^F{}_I \\ \hdashline Q^F{}_{II} \end{array}\right]$$

Reaction

Support Boundary Condition





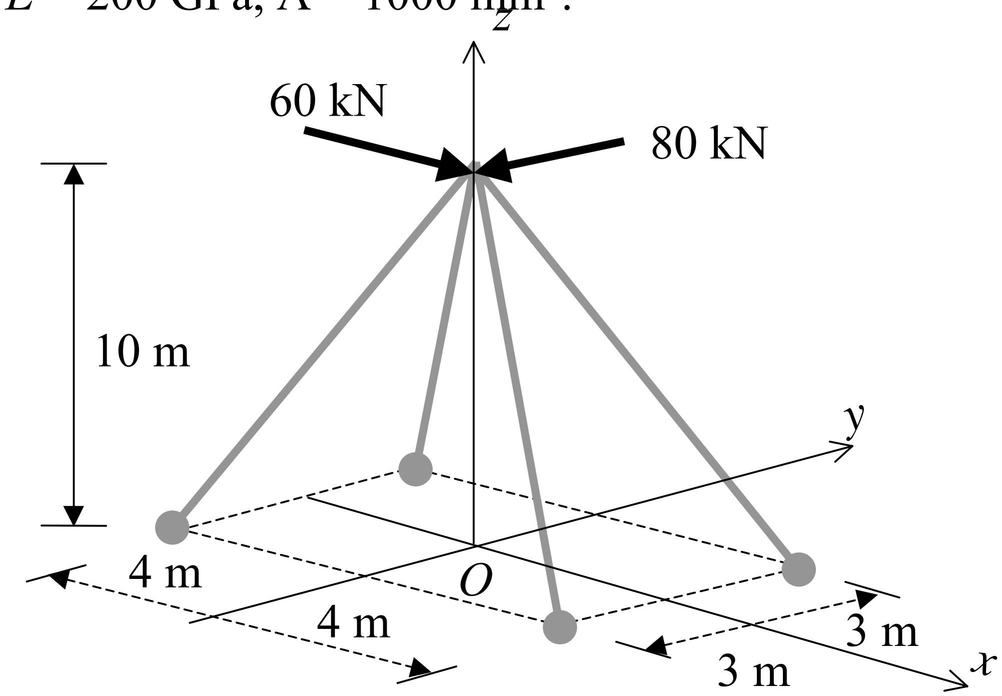
$$\begin{pmatrix} q_{ix} \\ q_{iy} \\ q_{iz} \\ q_{jx} \\ q_{jy} \\ q_{jz} \end{pmatrix} = \frac{EA}{L} \begin{pmatrix} \lambda_x\lambda_x & \lambda_x\lambda_y & \lambda_x\lambda_z & -\lambda_x\lambda_x & -\lambda_x\lambda_y & -\lambda_x\lambda_z \\ \lambda_y\lambda_x & \lambda_y\lambda_y & \lambda_y\lambda_z & -\lambda_y\lambda_x & -\lambda_y\lambda_y & -\lambda_y\lambda_z \\ \lambda_z\lambda_x & \lambda_z\lambda_y & \lambda_z\lambda_z & -\lambda_z\lambda_x & -\lambda_z\lambda_y & -\lambda_z\lambda_z \\ -\lambda_x\lambda_x & -\lambda_x\lambda_y & -\lambda_x\lambda_z & \lambda_x\lambda_x & \lambda_x\lambda_y & \lambda_x\lambda_z \\ -\lambda_y\lambda_x & -\lambda_y\lambda_y & -\lambda_y\lambda_z & \lambda_y\lambda_x & \lambda_y\lambda_y & \lambda_y\lambda_z \\ -\lambda_z\lambda_x & -\lambda_z\lambda_y & -\lambda_z\lambda_z & \lambda_z\lambda_x & \lambda_z\lambda_y & \lambda_z\lambda_z \end{pmatrix} \begin{pmatrix} d_{ix} \\ d_{iy} \\ d_{iz} \\ d_{jx} \\ d_{jy} \\ d_{jz} \end{pmatrix}$$

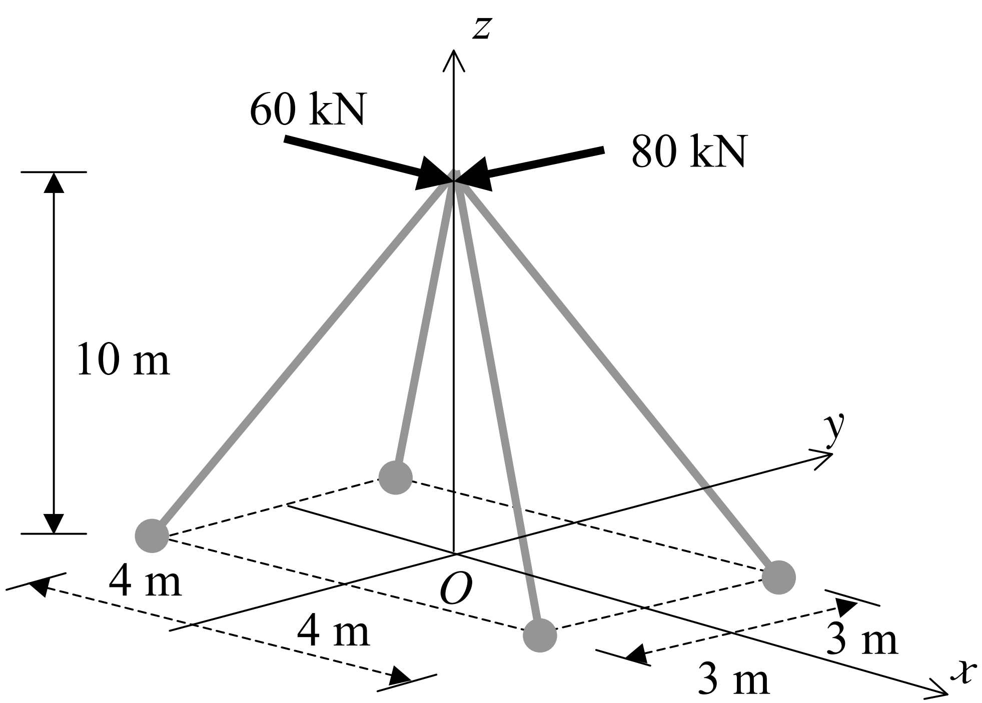
$$\begin{bmatrix} q'_i \\ q'_j \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} d'_i \\ d'_j \end{bmatrix} + \begin{bmatrix} q'^F_i \\ q'^F_j \end{bmatrix}$$

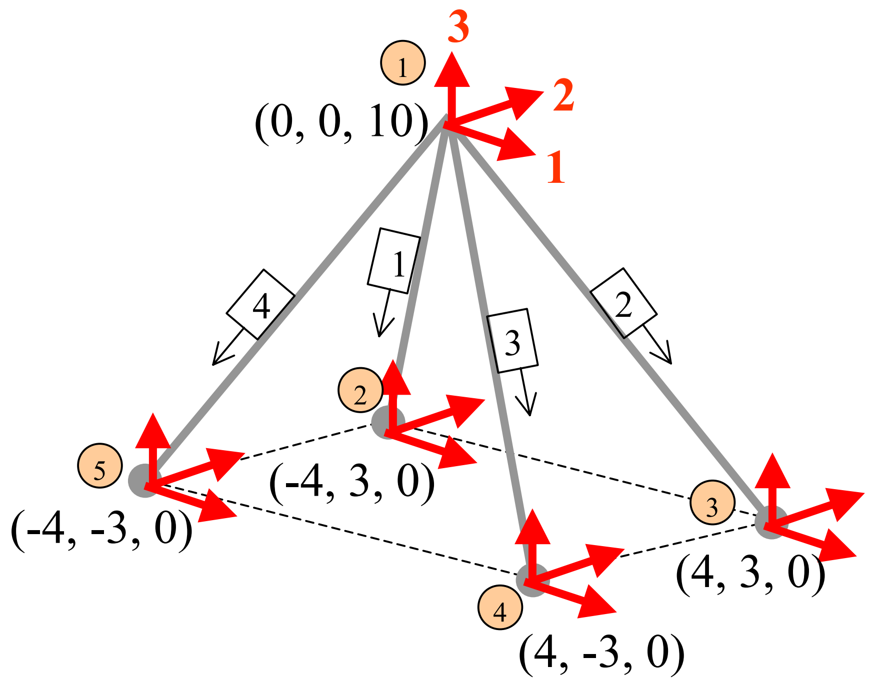
$$[q'_j]_m = \frac{EA}{L} \begin{bmatrix} -\lambda_x & -\lambda_y & -\lambda_z & \lambda_x & \lambda_y & \lambda_z \end{bmatrix} \begin{bmatrix} d_{ix} \\ d_{iy} \\ d_{iz} \\ d_{jx} \\ d_{jy} \\ d_{jz} \end{bmatrix} + [q'^F]$$

Example 6

For the truss shown, use the stiffness method to:
(b) Determine the **end forces** of each member.
(a) Determine the **deflections** of the loaded joint.
Take E = 200 GPa, A = 1000 mm².







$$\lambda_m = \lambda_x\hat{i} + \lambda_y\hat{j} + \lambda_z\hat{k}$$

$$\lambda_1 = (-4/11.18)\hat{i} + (3/11.18)\hat{j} + (-10/11.18)\hat{k}$$
$$= -0.3578\,\hat{i} + 0.2683\,\hat{j} - 0.8944\,\hat{k}$$

$$\lambda_2 = (+4/11.18)\hat{i} + (3/11.18)\hat{j} + (-10/11.18)\hat{k}$$
$$= +0.3578\,\hat{i} + 0.2683\,\hat{j} - 0.8944\,\hat{k}$$

$$\lambda_3 = (+4/11.18)\hat{i} + (-3/11.18)\hat{j} + (-10/11.18)\hat{k}$$
$$= +0.3578\,\hat{i} - 0.2683\,\hat{j} - 0.8944\,\hat{k}$$

$$\lambda_4 = (-4/11.18)\hat{i} + (-3/11.18)\hat{j} + (-10/11.18)\hat{k}$$
$$= -0.3578\,\hat{i} - 0.2683\,\hat{j} - 0.8944\,\hat{k}$$

Member	λ_x	λ_y	λ_z
#1	-0.3578	+0.2683	-0.8944
#2	+0.3578	+0.2683	-0.8944
#3	+0.3578	-0.2683	-0.8944
#4	-0.3578	-0.2683	-0.8944

Member	λ_x	λ_y	λ_z
#1	-0.3578	+0.2683	-0.8944
#2	+0.3578	+0.2683	-0.8944
#3	+0.3578	-0.2683	-0.8944
#4	-0.3578	-0.2683	-0.8944

$$[k_{11}]_1 = \frac{AE}{L} \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} +0.128 & -0.096 & +0.320 \\ -0.096 & +0.072 & -0.240 \\ +0.320 & -0.240 & +0.80 \end{bmatrix} \end{array}$$

$$[k_{11}]_2 = \frac{AE}{L} \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} +0.128 & +0.096 & -0.320 \\ +0.096 & +0.072 & -0.240 \\ -0.320 & -0.240 & +0.80 \end{bmatrix} \end{array}$$

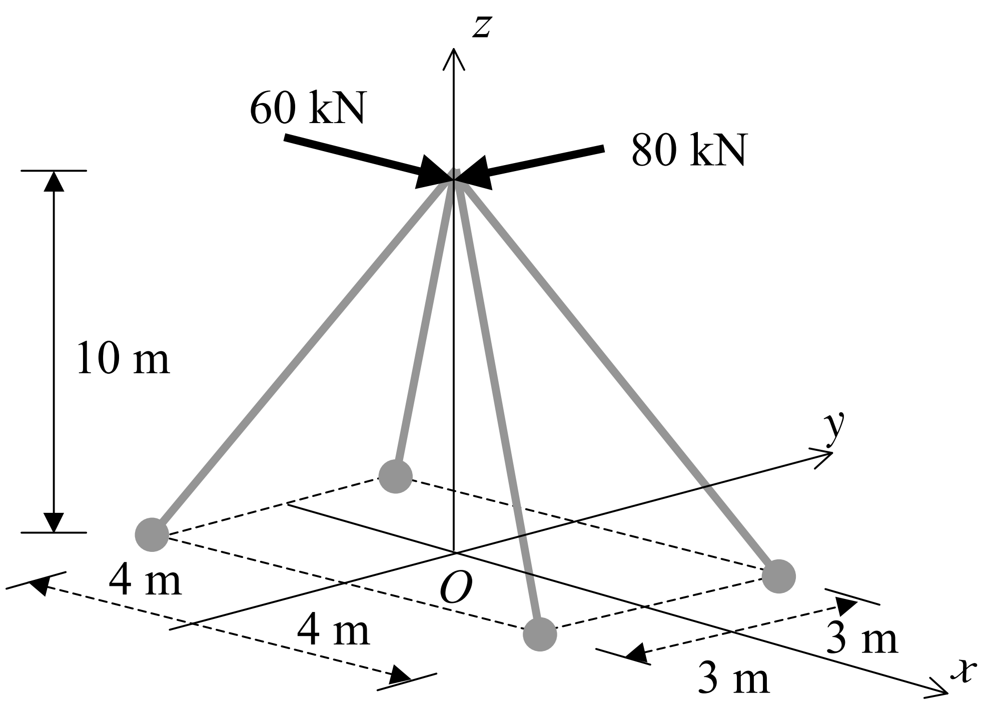
Member Stiffness Matrix $[k]_{6x6}$

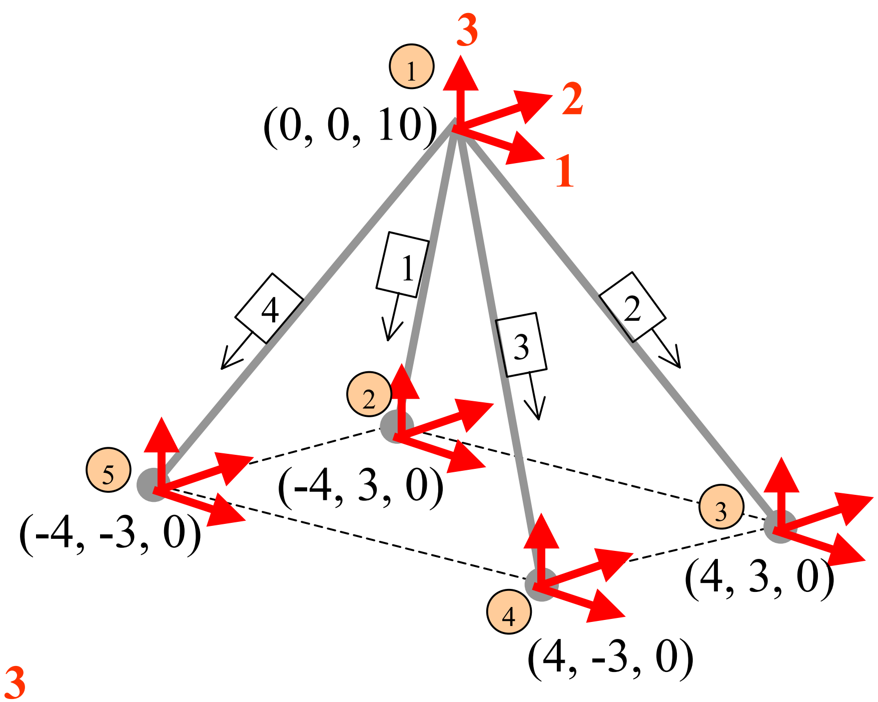
$$[k]_m = \begin{bmatrix} [k_{11}]_{3x3} & [k_{12}]_{3x3} \\ [k_{21}]_{3x3} & [k_{22}]_{3x3} \end{bmatrix}$$

$$[k_{11}]_3 = \frac{AE}{L} \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} +0.128 & -0.096 & -0.320 \\ -0.096 & +0.072 & +0.240 \\ -0.320 & +0.240 & +0.80 \end{bmatrix} \end{array}$$

$$[k_{11}]_4 = \frac{AE}{L} \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} +0.128 & +0.096 & +0.320 \\ +0.096 & +0.072 & +0.240 \\ +0.320 & +0.240 & +0.80 \end{bmatrix} \end{array}$$

$$[K_{I,I}] = \frac{AE}{L} \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.512 & 0.0 & 0.0 \\ 0.0 & 0.288 & 0.0 \\ 0.0 & 0.0 & 3.2 \end{bmatrix} \end{array}$$





$$[Q] = [K][D] + [Q^F]$$

$$\begin{pmatrix} 60 \\ -80 \\ 0.0 \end{pmatrix} = \frac{AE}{L} \begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 0.512 & 0.0 & 0.0 \\ 0.0 & 0.288 & 0.0 \\ 0.0 & 0.0 & 3.2 \end{pmatrix} \end{array} \begin{pmatrix} D_1 \\ D_2 \\ D_3 \end{pmatrix} + \begin{pmatrix} 0.0 \\ 0.0 \\ 0.0 \end{pmatrix}$$

Global equilibrium matrix:

$$[Q] = [K][D] + [Q^F]$$

Joint Load — Unknown Displacement — Fixed End Forces

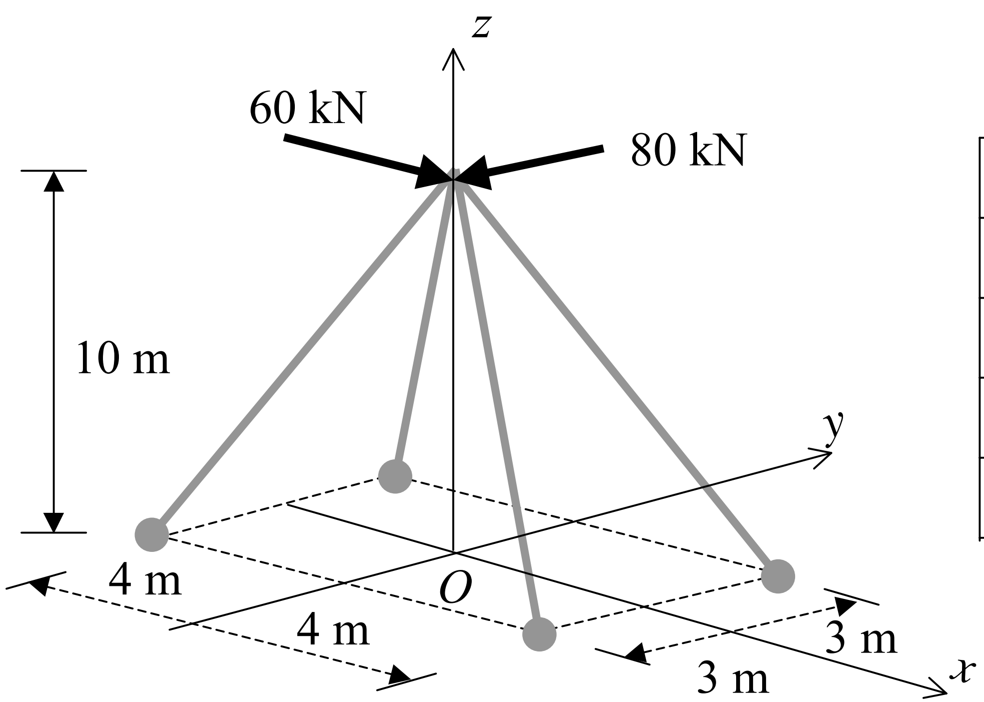
$$\begin{bmatrix} Q_I \\ \hline Q_{II} \end{bmatrix} = \left[\begin{array}{c|c} \mathbf{K_{I,I}} & \mathbf{K_{I,II}} \\ \hline \mathbf{K_{II,I}} & \mathbf{K_{II,II}} \end{array}\right] \begin{bmatrix} D_u \\ \hline D_k \end{bmatrix} + \begin{bmatrix} Q^F{}_I \\ \hline Q^F{}_{II} \end{bmatrix}$$

Reaction — Support Boundary Condition

$(AE/L) = (1\text{x}10^{-3})(200\text{x}10^{6})/(11.18) = 17.89\text{x}10^{3}$ kN

$$\begin{bmatrix} 60 \\ -80 \\ 0.0 \end{bmatrix} = \frac{AE}{L} \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.512 & 0.0 & 0.0 \\ 0.0 & 0.288 & 0.0 \\ 0.0 & 0.0 & 3.2 \end{bmatrix} \end{array} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} + \begin{bmatrix} 0.0 \\ 0.0 \\ 0.0 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \frac{L}{AE} \begin{bmatrix} +117.2 \\ -277.8 \\ 0.0 \end{bmatrix} = \begin{bmatrix} 6.551 & \text{mm} \\ -15.53 & \text{mm} \\ 0.0 & \text{mm} \end{bmatrix}$$



Member	λ_x	λ_y	λ_z
#1	-0.3578	+0.2683	-0.8944
#2	+0.3578	+0.2683	-0.8944
#3	+0.3578	-0.2683	-0.8944
#4	-0.3578	-0.2683	-0.8944

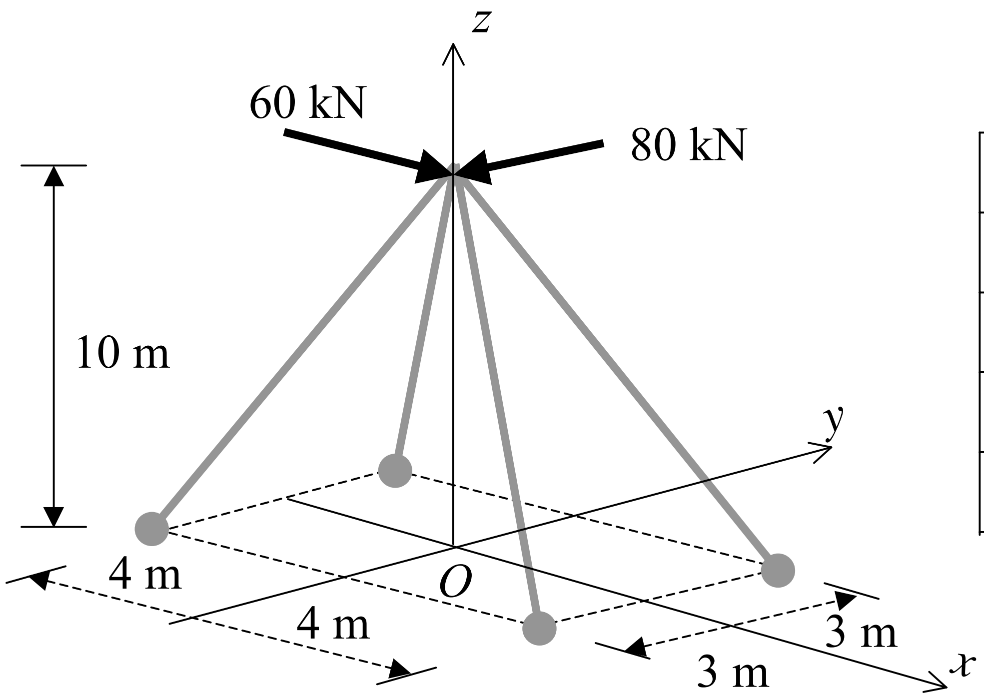
Member forces:

$$[q'_j]_m = \frac{AE}{L}\begin{bmatrix} -\lambda_x & -\lambda_y & -\lambda_z & \lambda_x & \lambda_y & \lambda_z \end{bmatrix}\begin{pmatrix} d_{xi} \\ d_{yi} \\ d_{zi} \\ d_{xj} \\ d_{yj} \\ d_{zj} \end{pmatrix} + \begin{bmatrix} q'^F \end{bmatrix}$$

$$\begin{pmatrix} d_{xi} \\ d_{yi} \\ d_{zi} \end{pmatrix} \to \begin{pmatrix} D_1 \\ D_2 \\ D_3 \end{pmatrix}, \quad \begin{pmatrix} d_{xj} \\ d_{yj} \\ d_{zj} \end{pmatrix} \to [\,0\,]$$

$$[q'_j]_1 = \frac{AE}{L}\begin{bmatrix} +0.3578 & -0.2683 & +0.8944 \end{bmatrix}\frac{L}{AE}\begin{pmatrix} 117.2 \\ -277.8 \\ 0.0 \end{pmatrix}$$

$$= +116.5 \text{ kN} \quad \text{(T)}$$



Member	λ_x	λ_y	λ_z
#1	-0.3578	+0.2683	-0.8944
#2	+0.3578	+0.2683	-0.8944
#3	+0.3578	-0.2683	-0.8944
#4	-0.3578	-0.2683	-0.8944

$$[q'_{j}]_2 = \frac{AE}{L}\begin{bmatrix} -0.3578 & -0.2683 & +0.8944 \end{bmatrix} \frac{L}{AE}\begin{pmatrix} 117.2 \\ -277.8 \\ 0.0 \end{pmatrix} = +32.61 \text{ kN} \quad \text{(T)}$$

$$[q'_{j}]_3 = \frac{AE}{L}\begin{bmatrix} -0.3578 & +0.2683 & +0.8944 \end{bmatrix} \frac{L}{AE}\begin{pmatrix} 117.2 \\ -277.8 \\ 0.0 \end{pmatrix} = -116.5 \text{ kN} \quad \text{(T)}$$

$$[q'_{j}]_4 = \frac{AE}{L}\begin{bmatrix} +0.3578 & +0.2683 & +0.8944 \end{bmatrix} \frac{L}{AE}\begin{pmatrix} 117.2 \\ -277.8 \\ 0.0 \end{pmatrix} = -32.61 \text{ kN} \quad \text{(T)}$$

Member	λ_x	λ_y	λ_z	$[q'_j]_m$
#1	-0.3578	+0.2683	-0.8944	116.5
#2	+0.3578	+0.2683	-0.8944	32.6
#3	+0.3578	-0.2683	-0.8944	-116.5
#4	-0.3578	-0.2683	-0.8944	-32.6