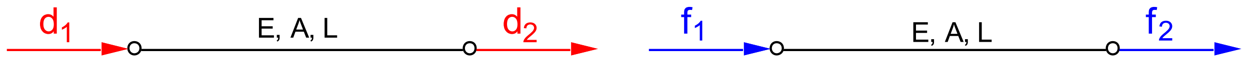
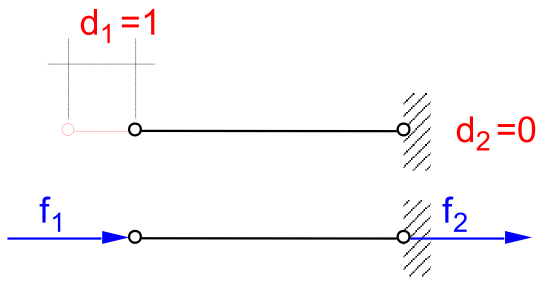
YATAY DURAN BİR KAFES ÇUBUĞUNUN RİJİTLİK MATRİSİ



1. Durum: $d_1=1$, $d_2=0$



$$f_1 = \frac{EA}{L}$$

$$\sum F_x = 0 \Rightarrow f_2 = -f_1 = -\frac{EA}{L}$$

2. Durum: $d_1=0$, $d_2=1$

$$f_2 = \frac{EA}{L}$$

$$\sum F_x = 0 \Rightarrow f_1 = -f_2 = -\frac{EA}{L}$$

Matris formunda yazarsak

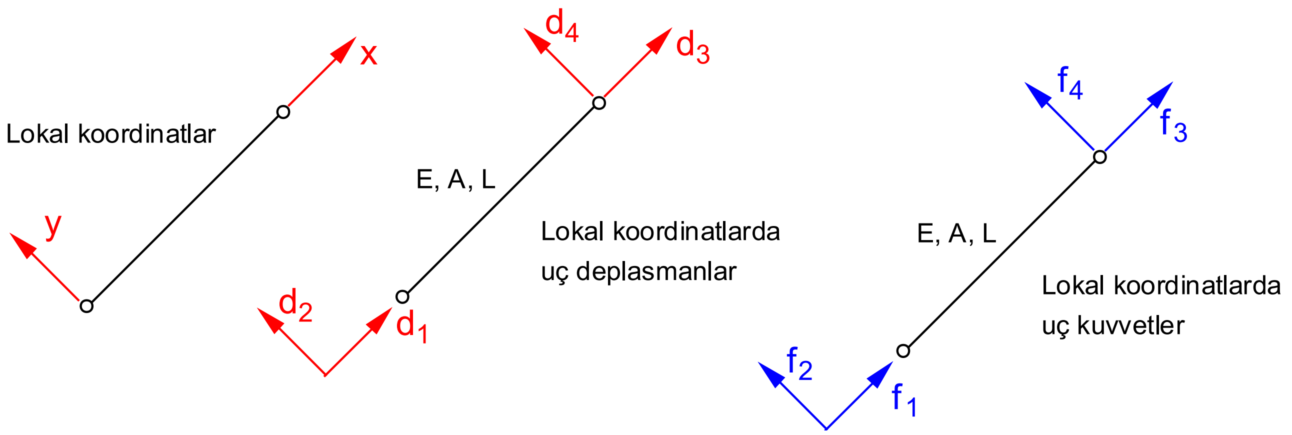
$$\{f\} = [\hat{k}]\{d\}$$

$$\begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix} = \begin{bmatrix} \frac{EA}{L} & -\frac{EA}{L} \\ -\frac{EA}{L} & \frac{EA}{L} \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \end{Bmatrix} \quad \textbf{veya} \quad \begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix} = \frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \end{Bmatrix}$$

Görüldüğü gibi rijitlik matrisi simetrik bir kare matristir.

EĞİK DURAN BİR KAFES ÇUBUĞUNUN RİJİTLİK MATRİSİ

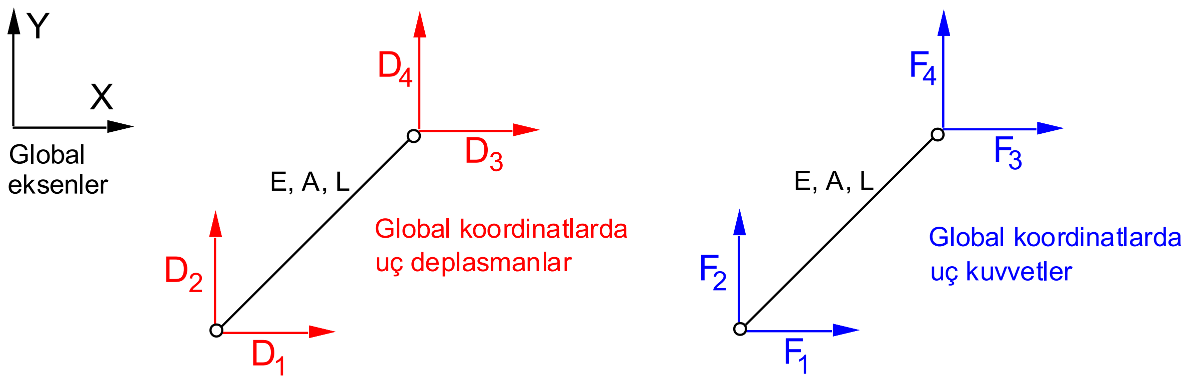
Lokal Eksenlerdeki Durum



$$\{f\} = [\hat{k}]\{d\}$$

$$\begin{Bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{Bmatrix} = \frac{EA}{L}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{Bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{Bmatrix}$$

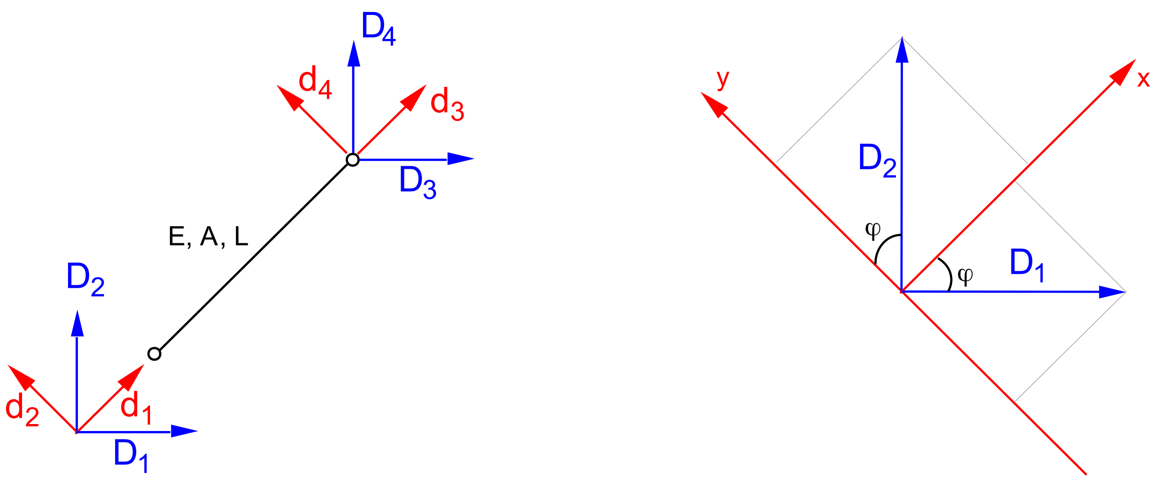
Global Eksenlerdeki Durum



$$\{F\} = [k]\{D\}$$

Buradaki [k] yı lokal eksenlerden global eksenlere dönüştürme işlemi ile elde edelim.

Global koordinatlardaki uç deplasmanların lokal koordinatlara dönüştürülmesi



$$d_1 = D_1 \cdot \cos\varphi + D_2 \cdot \sin\varphi$$
$$d_2 = -D_1 \cdot \sin\varphi + D_2 \cdot \cos\varphi$$
$$d_3 = D_3 \cdot \cos\varphi + D_4 \cdot \sin\varphi$$
$$d_4 = -D_3 \cdot \sin\varphi + D_4 \cdot \cos\varphi$$
$$s = \sin\varphi \qquad c = \cos\varphi$$

$$\begin{Bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{Bmatrix} = \begin{bmatrix} c & s & 0 & 0 \\ -s & c & 0 & 0 \\ 0 & 0 & c & s \\ 0 & 0 & -s & c \end{bmatrix} \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{Bmatrix}$$

$\{d\} = [T]\{D\}$ ve benzer şekilde $\{f\} = [T]\{F\}$ olur.

$\{f\} = [\hat{k}]\{d\}$ olduğunu biliyoruz. Buradan,

$$[T]\{F\} = [\hat{k}][T]\{D\}$$

yazarsak ve her iki tarafı $[T]^T$ ile çarparsak,

$$[T]^T[T]\{F\} = [T]^T[\hat{k}][T]\{D\}$$

olur.

$[T]^T[T] = [I]$(birim matristir)

$\{F\} = [T]^T[\hat{k}][T]\{D\}$ olur.

$\{F\} = [k]\{D\}$ olduğunu söylemiştik. Böylece,

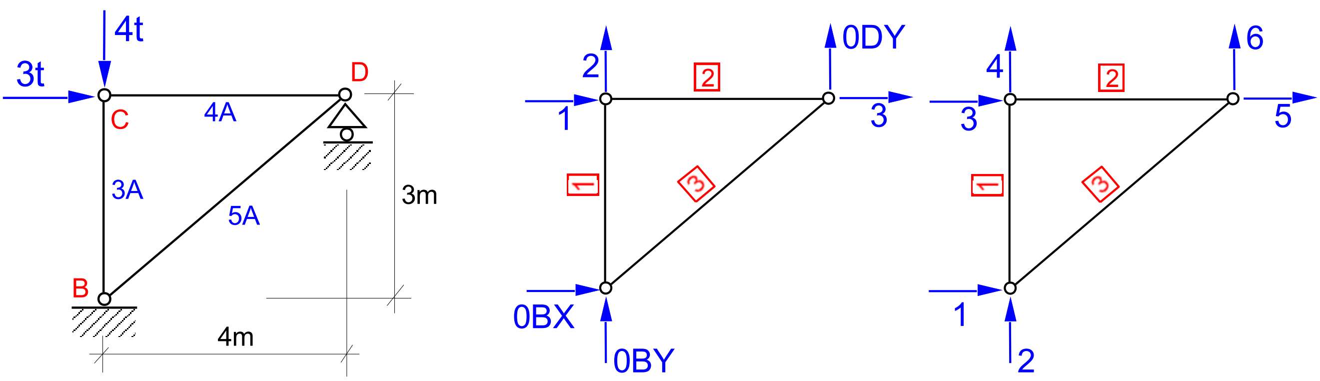
$$[k] = [T]^T[\hat{k}][T]$$

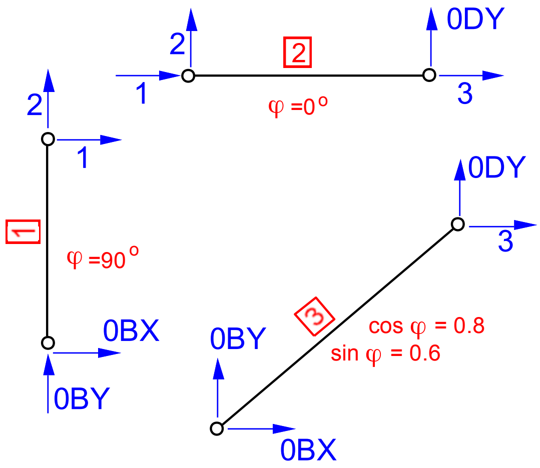
olacaktır.

Matlab ile işlemleri yaparsak:

```
>> syms c s
>> T=[c s 0 0; -s c 0 0; 0 0 c s; 0 0 -s c]
>> k_local=[1 0 -1 0; 0 0 0 0; -1 0 1 0; 0 0 0 0]
>> k_global=T.'*k_local*T
k_global =
[   c^2,   c*s, -c^2, -c*s]
[   c*s,   s^2, -c*s, -s^2]
[ -c^2, -c*s,   c^2,   c*s]
[ -c*s, -s^2,   c*s,   s^2]
>> pretty(ans)
[   2                2            ]
[  c        c s      -c       -c s]
[                                 ]
[            2                 2  ]
[c s        s        -c s     -s  ]
[                                 ]
[   2                2            ]
[-c        -c s      c        c s ]
[                                 ]
[            2                 2  ]
[-c s      -s        c s       s  ]
```

$$[k]=\frac{EA}{L}\begin{bmatrix} c^2 & c\cdot s & -c^2 & -c\cdot s \\ c\cdot s & s^2 & -c\cdot s & -s^2 \\ -c^2 & -c\cdot s & c^2 & c\cdot s \\ -c\cdot s & -s^2 & c\cdot s & s^2 \end{bmatrix}$$





$$[k]_1 = EA\begin{array}{c} \\ \\ \\ \\ \end{array}\begin{array}{cccc|c} 0BX & 0BY & 1 & 2 & \\ \hline 0 & 0 & 0 & 0 & 0BX \\ 0 & 1 & 0 & -1 & 0BY \\ 0 & 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 1 & 2 \end{array}$$

$$[k]_2 = EA\begin{array}{cccc|c} 1 & 2 & 3 & 0DY & \\ \hline 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 2 \\ -1 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0DY \end{array}$$

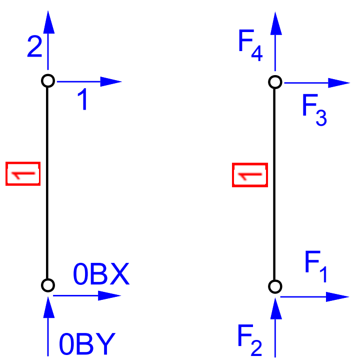
$$[k]_3 = EA\begin{array}{cccc|c} 0BX & 0BY & 3 & 0DY & \\ \hline 0.64 & 0.48 & -0.64 & -0.48 & 0BX \\ 0.48 & 0.36 & -0.48 & -0.36 & 0BY \\ -0.64 & -0.48 & 0.64 & 0.48 & 3 \\ -0.48 & -0.36 & 0.48 & 0.36 & 0DY \end{array}$$

$$[K] = EA\begin{array}{ccc|c} 1 & 2 & 3 & \\ \hline 1 & 0 & -1 & 1 \\ 0 & 1 & 0 & 2 \\ -1 & 0 & 1.64 & 3 \end{array} \qquad \{F\} = \begin{Bmatrix} 3 \\ -4 \\ 0 \end{Bmatrix}\begin{matrix} 1 \\ 2 \\ 3 \end{matrix}$$

$$\{D\} = [K]^{-1}\{F\}$$

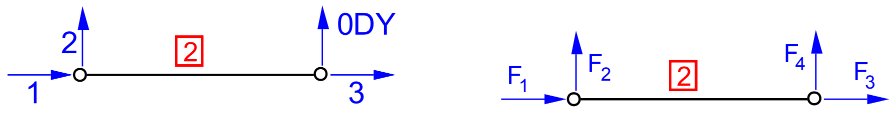
$$\{D\} = \frac{1}{EA}\begin{Bmatrix} 7.6875 \\ -4 \\ 4.6875 \end{Bmatrix}\begin{matrix} 1 \\ 2 \\ 3 \end{matrix}$$

Eleman Uç Kuvvetleri

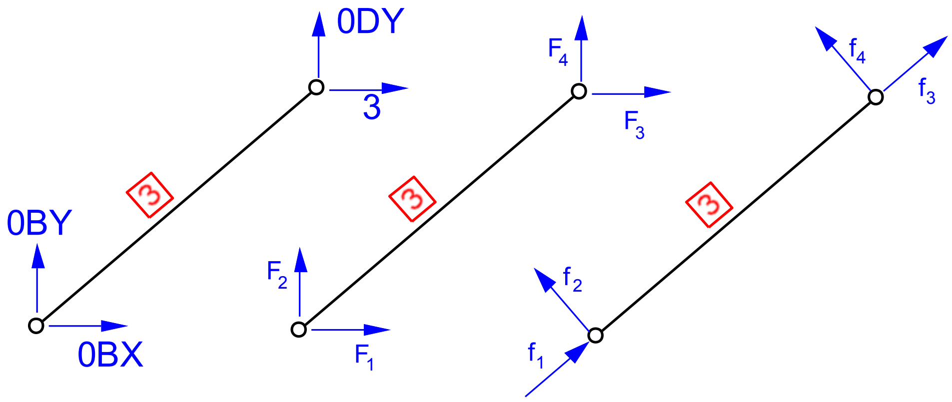


$$\{F\} = [k]\{D\}$$

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix} = EA \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 7.6875 \\ -4 \end{Bmatrix} \frac{1}{EA} = \begin{Bmatrix} 0 \\ 4 \\ 0 \\ -4 \end{Bmatrix}$$



$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix} = EA \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} 7.6875 \\ -4 \\ 4.6875 \\ 0 \end{Bmatrix} \frac{1}{EA} = \begin{Bmatrix} 3 \\ 0 \\ -3 \\ 0 \end{Bmatrix}$$



$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix} = EA \begin{bmatrix} 0.64 & 0.48 & -0.64 & -0.48 \\ 0.48 & 0.36 & -0.48 & -0.36 \\ -0.64 & -0.48 & 0.64 & 0.48 \\ -0.48 & -0.36 & 0.48 & 0.36 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 4.6875 \\ 0 \end{Bmatrix} \frac{1}{EA} = \begin{Bmatrix} -3 \\ -2.25 \\ 3 \\ 2.25 \end{Bmatrix}$$

$$\begin{Bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{Bmatrix} = \begin{bmatrix} 0.8 & 0.6 & 0 & 0 \\ -0.6 & 0.8 & 0 & 0 \\ 0 & 0 & 0.8 & 0.6 \\ 0 & 0 & -0.6 & 0.8 \end{bmatrix} \begin{Bmatrix} -3 \\ -2.25 \\ 3 \\ 2.25 \end{Bmatrix} = \begin{Bmatrix} -3.75 \\ 0 \\ 3.75 \\ 0 \end{Bmatrix}$$

Mesnet Reaksiyonları

$$\begin{Bmatrix} BX \\ BY \\ DY \end{Bmatrix} = EA \begin{bmatrix} \overset{1}{0} & \overset{2}{0} & \overset{3}{-0.64} \\ 0 & -1 & -0.48 \\ 0 & 0 & 0.48 \end{bmatrix} \begin{matrix} 0BX \\ 0BY \\ 0DY \end{matrix} \begin{Bmatrix} 7.6875 \\ -4 \\ 4.6875 \end{Bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \frac{1}{EA} = \begin{Bmatrix} -3 \\ 1.75 \\ 2.25 \end{Bmatrix}$$